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Finite difference schemes for linear stochastic integro-differential equations

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Abstract

We study the rate of convergence of an explicit and an implicit–explicit finite difference scheme for linear stochastic integro-differential equations of parabolic type arising in non-linear filtering of jump–diffusion processes. We show that the rate is of order one in space and order one-half in time.

Keywords: Stochastic integro-differential equations; Finite differences; Lévy processes

1. Introduction

Let $(\Omega, \mathcal{F}, \mathbf{F}, P)$, $\mathbf{F} = (\mathcal{F}_t)_{t\geq 0}$, be a complete filtered probability space such that the filtration is right continuous and $\mathcal{F}_0$ contains all P-null sets of $\mathcal{F}$. Let $\{w^\varrho\}_{\varrho=1}^\infty$ be a sequence of independent real-valued $\mathbf{F}$-adapted Wiener processes. Let $\pi_1(dz)$ and $\pi_2(dz)$ be Borel sigma-finite measures on $\mathbf{R}^d$ satisfying

$$\int_{\mathbf{R}^d} |z|^2 \wedge 1 \, \pi_r(dz) < \infty, \quad r \in \{1, 2\}.$$

Let $q(dt, dz) = p(dt, dz) - \pi_2(dz)dt$ be a compensated $\mathbf{F}$-adapted Poisson random measure on $\mathbf{R}_+ \times \mathbf{R}^d$. Let $T > 0$ be an arbitrary fixed constant. On $[0, T] \times \mathbf{R}^d$, we consider finite difference

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approximations for the following stochastic integro-differential equation (SIDE)

$$du_t = ((\mathcal{L}_t + I)u_t + f_t)\,dt + \sum_{\varrho=1}^{\infty} \left(\mathcal{N}_t^{\varrho} u_t + g_t^{\varrho}\right) dw_t^{\varrho} + \int_{\mathbf{R}^d} (\mathcal{I}(z)u_{t-} + o_t(z))\, q(dt, dz), \tag{1.1}$$

with initial condition

$$u_0(x) = \varphi(x), \quad x \in \mathbf{R}^d,$$

where the operators are given by

$$\begin{aligned}
\mathcal{L}_t\phi(x) &:= \sum_{i,j=0}^{d} a_t^{ij}(x)\partial_{ij}\phi(x),\\
I\phi(x) &:= \int_{\mathbf{R}^d} \left(\phi(x+z) - \phi(x) - \mathbf{1}_{[-1,1]}(|z|)\sum_{j=1}^{d} z_j\partial_j\phi(x)\right)\pi_1(dz), \qquad (1.2)\\
\mathcal{N}_t^{\varrho}\phi(x) &:= \sum_{i=0}^{d} \sigma_t^{i\varrho}(x)\partial_i\phi(x), \quad \mathcal{I}(z)\phi(x) = \phi(x+z) - \phi(x).
\end{aligned}$$

Here $\partial_i = \frac{\partial}{\partial x_i}$ for $i = 1, 2, \ldots, d$, ∂_0 is the identity operator, and $\partial_{ij} = \partial_i\partial_j$, for $i, j = 0, 1, 2, \ldots, d$.

Eq. (1.1) arises naturally in non-linear filtering of jump–diffusion processes. We refer the reader to [4,5] for more information about non-linear filtering of jump–diffusions and the derivation of the Zakai equation. Various methods have been proposed to solve stochastic partial differential equations (SPDEs) numerically. For SPDEs driven by continuous martingale noise see, for example, [3,7,8,13,21,16,23] and for SPDEs driven by discontinuous martingale noise, see [15,14,20,1]. Among the various methods considered in the literature is the method of finite differences. For second order linear SPDEs driven by continuous martingale noise it is well-known that the $L^p(\Omega)$-pointwise error of approximation in space is proportional to the parameter h of the finite difference (see, e.g., [24]). In [13], I. Gyöngy and A. Millet consider abstract discretization schemes for stochastic evolution equations driven by continuous martingale noise in the variational framework and, as a particular example, show that the $L^2(\Omega)$-pointwise rate of convergence of an Euler–Maruyama (explicit and implicit) finite difference scheme is of order one in space and one-half in time. More recently, it was shown by I. Gyöngy and N.V. Krylov that under certain regularity conditions, the rate of convergence in space of a semi-discretized finite difference approximation of a linear second order SPDE driven by continuous martingale noise can be accelerated to any order by Richardson's extrapolation method. For the non-degenerate case, we refer to [10,11], and for the degenerate case, we refer to [9]. In [17,18], E. Hall proved that the same method of acceleration can be applied to implicit time-discretized SPDEs driven by continuous martingale noise.

In the literature, finite element, spectral, and, more generally, Galerkin schemes have been studied for SPDEs driven by discontinuous martingale noise. One of the earliest works in this direction is a paper [15] by E. Hausenblas and I. Marchis concerning $L^p(\Omega)$-convergence of Galerkin approximation schemes for abstract stochastic evolution equations in Banach spaces driven by Poisson noise of impulsive-type. As an application of their result, they study a spectral approximation of a linear SPDE in $L^2([0, 1])$ with Neumann boundary conditions driven