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Corrigendum

Corrigendum to "Phase transition in equilibrium fluctuations of symmetric slowed exclusion" [Stochastic Process. Appl. 123(12) (2013) 4156–4185]

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Abstract

We present the correct space of test functions for the Ornstein–Uhlenbeck processes defined in Franco et al. (2013). Under these new spaces, an invariance with respect to a second order operator is shown, granting the existence and uniqueness of those processes. Moreover, we detail how to prove some properties of the semigroups, which are required in the proof of uniqueness.

1. Outline

In our paper [2] it was used that, if $H \in \mathcal{S}_\beta(\mathbb{R})$, then

$$\Delta_\beta T_t^\beta H \in \mathcal{S}_\beta(\mathbb{R}), \tag{1.1}$$

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which is not true. Above, Δ_β is essentially the Laplacian operator, but defined in the space of test functions $\mathcal{S}_\beta(\mathbb{R})$ and the operator T_t^β is the semigroup of the related PDE. The reader can find the complete definitions in [2].

To clarify ideas, for $\beta \in [0, 1)$, $\mathcal{S}_\beta(\mathbb{R})$ is the classical Schwartz space, and T_t^β is the semigroup of the heat equation. It is well known (via Fourier transform, see [4] for instance) that the heat equation preserves the Schwartz space. Moreover, any derivative of a function in the Schwartz space is again in the Schwartz space. Therefore, condition (1.1) is true for $\beta \in [0, 1)$. However, for $\beta \geq 1$ it is not. In this note we properly redefine $\mathcal{S}_\beta(\mathbb{R})$ for $\beta \geq 1$, so that (1.1) becomes true for any $\beta \geq 0$.

A remark: after the statement of Proposition 3.2 of the present article we give the precise locations where (1.1) is used in [2]. One of these locations is the proof of uniqueness of the corresponding generalized Ornstein–Uhlenbeck processes (O.U.). There are some recent works on generalized O.U. processes whose associated operators do not satisfy (1.1), see for example [1]. However, such topic is quite technical and hard to deal with. Thus, we have chosen to modify the definition of test functions in order to have (1.1), so that the proof of uniqueness, presented in [2], is valid.

Finally, some details missing in the original paper are also included here.

2. Introduction

The purpose of these notes is to present a correction in the definition of the space of test functions of the Ornstein–Uhlenbeck processes which govern the equilibrium density fluctuations of symmetric simple exclusion processes with a slow bond as defined in [2], that we denote by $\{\eta_{tn^2} : t \geq 0\}$. We consider the processes starting from the invariant state, namely the Bernoulli product measure of parameter ρ. Fix $\rho \in (0, 1)$. We recall now the notion of the density fluctuation field as the linear functional acting on test functions $H \in \mathcal{S}_\beta(\mathbb{R})$ as

$$\mathcal{Y}_t^n(H) = \frac{1}{\sqrt{n}} \sum_{x \in \mathbb{Z}} H\Big(\frac{x}{n}\Big)(\eta_{tn^2} - \rho),$$

see [2, Subsection 2.5]. The limiting process of $\mathcal{Y}_t^n$, denoted by $\mathcal{Y}_t$, is a generalized Ornstein–Uhlenbeck process solution of

$$d\mathcal{Y}_t = \Delta_\beta \mathcal{Y}_t dt + \sqrt{2\chi(\rho)}\nabla_\beta d\mathcal{W}_t, \tag{2.1}$$

as explained in [2], where $\chi(\rho) = \rho(1-\rho)$, the stochastic process $\mathcal{W}_t$ is an $\mathcal{S}'_\beta(\mathbb{R})$-valued Brownian motion, and Δ_β and ∇_β are properly defined below.

3. The space of test functions

We start by redefining the space of test functions $\mathcal{S}_\beta(\mathbb{R})$ where the operators Δ_β and ∇_β are defined.

3.1. Definition of $\mathcal{S}_\beta(\mathbb{R})$ and the operators Δ_β and ∇_β

Definition 1. For $\beta \in [0, +\infty]$ and $\alpha > 0$, let $L^2_\beta(\mathbb{R})$ be the space of functions $H : \mathbb{R} \to \mathbb{R}$ with $\|H\|^2_{2,\beta} := \|H\|_2^2 + \frac{1}{\alpha^2}\mathbf{1}_{\beta=1}(H(0))^2 < +\infty$, where $\|H\|_2^2 = \int_{\mathbb{R}} H(u)^2 du$.