Available online at www.sciencedirect.com

ScienceDirect

Stochastic Processes and their Applications 126 (2016) 2634–2664

stochastic processes and their applications

www.elsevier.com/locate/spa

Exact convergence rates in central limit theorems for a branching random walk with a random environment in time☆

Zhiqiang Gao[a], Quansheng Liu[b,c,*]

[a] *School of Mathematical Sciences, Laboratory of Mathematics and Complex Systems, Beijing Normal University, Beijing 100875, China*

[b] *Univ. Bretagne-Sud, CNRS UMR 6205, LMBA, campus de Tohannic, F-56000 Vannes, France*

[c] *Changsha University of Science and Technology, School of Mathematics and Computing Science, Changsha 410004, China*

Received 12 December 2014; received in revised form 6 July 2015; accepted 25 February 2016
Available online 10 March 2016

Abstract

Chen (2001) derived exact convergence rates in a central limit theorem and a local limit theorem for a supercritical branching Wiener process. We extend Chen's results to a branching random walk under weaker moment conditions. For the branching Wiener process, our results sharpen Chen's by relaxing the second moment condition used by Chen to a moment condition of the form $\mathbb{E}X(\ln^+ X)^{1+\lambda} < \infty$. In the rate functions that we find for a branching random walk, we figure out some new terms which did not appear in Chen's work. The results are established in the more general framework, i.e. for a branching random walk with a random environment in time. The lack of the second moment condition for the offspring distribution and the fact that the exponential moment does not exist necessarily for the displacements make the proof

☆ The project is partially supported by the National Natural Science Foundation of China (NSFC, Grants No. 11101039, No. 11571052, No. 11271045, and No. 11401590), by a cooperation program between NSFC and CNRS of France (Grant No. 11311130103), by the Fundamental Research Funds for the Central Universities (2013YB52) and by the Natural Science Foundation of Guandong Province of China (Grant No. 2015A030313628).

* Corresponding author at: Univ. Bretagne-Sud, CNRS UMR 6205, LMBA, campus de Tohannic, F-56000 Vannes, France.

E-mail addresses: gaozq@bnu.edu.cn (Z. Gao), quansheng.liu@univ-ubs.fr (Q. Liu).

http://dx.doi.org/10.1016/j.spa.2016.02.013
0304-4149/

delicate; the difficulty is overcome by a careful analysis of martingale convergence using a truncating argument. The analysis is significantly more awkward due to the appearance of the random environment.

MSC: 60K37; 60J10; 60F05; 60J80

Keywords: Branching random walk; Random environment in time; Central limit theorems; Convergence rate

1. Introduction

The theory of branching random walk has been studied by many authors. It plays an important role, and is closely related to many problems arising in a variety of applied probability setting, including branching processes, multiplicative cascades, infinite particle systems, Quicksort algorithms and random fractals (see e.g. [31,32]). For recent developments of the subject, see e.g. Hu and Shi [23], Shi [38], Hu [22], Attia and Barral [4] and the references therein.

In the classical branching random walk, the point processes indexed by the particles u, formulated by the number of its children and their displacements, have a fixed constant distribution for all particles u. In reality this distributions may vary from generation to generation according to a random environment, just as in the case of a branching process in random environment introduced in [39,3,2]. In other words, the distributions themselves may be realizations of a stochastic process, rather than being fixed. This property makes the model be closer to the reality compared to the classical branching random walk. In this paper, we shall consider such a model, called *a branching random walk with a random environment in time*.

Different kinds of branching random walks in random environments have been introduced and studied in the literature. Baillon, Clément, Greven and den Hollander [6,19] considered the case where the offspring distribution of a particle situated at $z \in \mathbb{Z}^d$ depends on a random environment indexed by the location z, while the moving mechanism is controlled by a fixed deterministic law. Comets and Popov [12,13] studied the case where both the offspring distributions and the moving laws depend on a random environment indexed by the location. In the model studied in [9,24,34,41,14], the offspring distribution of a particle of generation n situated at $z \in \mathbb{Z}^d (d \geq 1)$ depends on a random space–time environment indexed by $\{(z, n)\}$, while each particle performs a simple symmetric random walk on d-dimensional integer lattice $\mathbb{Z}^d (d \geq 1)$. The model that we study in this paper is different from those mentioned above. It should also be mentioned that recently another different kind of branching random walks in time inhomogeneous environments has been considered extensively, see e.g. Fang and Zeitouni (2012, [16]), Zeitouni (2012, [42]) and Bovier and Hartung (2014, [10]). The readers may refer to these articles and references therein for more information.

Denote by $Z_n(\cdot)$ the counting measure which counts the number of particles of generation n situated in a given set. For the classical branching random walk, a central limit theorem on $Z_n(\cdot)$, first conjectured by Harris (1963, [21]), was shown by Asmussen and Kaplan (1976, [1,27]), and then extended to a general case by Klebaner (1982, [28]) and Biggins (1990, [7]); for a branching Wiener process, Révész (1994, [36]) studied the convergence rates in the central limit theorems and conjectured the exact convergence rates, which were confirmed by Chen (2001, [11]). Kabluchko (2012, [26]) generalized Chen's partial results using a different method. Révész, Rosen and Shi (2005, [37]) obtained a large time asymptotic expansion in the local limit theorem for branching Wiener processes, generalizing Chen's result.