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Max-stable random sup-measures with comonotonic tail dependence

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Abstract

Several objects in the Extremes literature are special instances of max-stable random sup-measures. This perspective opens connections to the theory of random sets and the theory of risk measures and makes it possible to extend corresponding notions and results from the literature with streamlined proofs. In particular, it clarifies the role of Choquet random sup-measures and their stochastic dominance property. Key tools are the LePage representation of a max-stable random sup-measure and the dual representation of its tail dependence functional. Properties such as complete randomness, continuity, separability, coupling, continuous choice, invariance and transformations are also analysed.

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1. Introduction

Random sup-measures provide a unified framework for dealing with a number of objects that naturally appear in the Extremes literature including temporal extremal processes [28,20],

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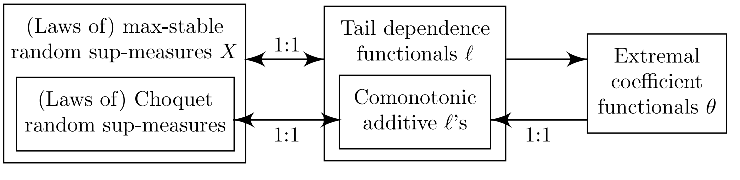
continuous choice models [29] or extremal loss in portfolios [41]; α-Fréchet sup-measures are the building blocks of max-stable processes [37]. In general, any stochastic process with upper semicontinuous paths can be viewed as a random sup-measure [28,27,39]. That is, the suprema of the process over sets yield a random sup-measure, while the values of the random sup-measure at singletons yield the upper semicontinuous process. The max-stability property of the process immediately translates into the same property of the random sup-measure.

Surprisingly, the notion of capacities and sup-measures has almost vanished from the theoretical developments on extremal processes over the past 20 years. This paper aims to clarify, extend and complement a number of results from the unifying perspective of sup-measures and capacities with streamlined proofs and connections to the theory of random sets and utility functions (or risk measures). The necessary preliminaries concerning capacities, random closed sets, random sup-measures, Choquet and extremal integrals are presented in Section 2.

Section 3 introduces *max-stable random sup-measures* X on a carrier space $\mathbb{E}$ and their *tail dependence functional* ℓ, which are the central objects in this paper. They are natural generalisations of max-stable random vectors and their (stable) tail dependence functions. For a given function f on $\mathbb{E}$, the tail dependence functional $\ell(f)$ characterises the distribution of the extremal integral of f with respect to X and so uniquely determines the distribution of the random sup-measure X. The values $\ell(\mathbb{1}_K)$ of ℓ on indicator functions $\mathbb{1}_K$ are called *extremal coefficients* and denoted by $\theta(K)$.

Generalising [31,30,25] we give a *complete characterisation* of the tail dependence functional as an upper semicontinuous homogeneous max-completely alternating functional and of the extremal coefficient functional as an upper semicontinuous union-completely alternating functional. Motivated by the family of stochastic processes studied in [38] and characterised by the fact that their distributions are in a one-to-one correspondence with extremal coefficient functionals, we identify the family of max-stable random sup-measures that have the same property. While in [38] such processes were called Tawn–Molchanov processes (TM processes), here we call their sup-measure analogues *Choquet random sup-measures (CRSMs)*. The key argument relies on the fact that the *comonotonic additivity* property of the tail dependence functional ℓ ensures that ℓ equals the Choquet integral with respect to θ, and so the distribution of the random sup-measure is uniquely determined by θ. This observation clarifies a number of properties of TM processes from [38] and establishes connections with the studies of coherent risk measures that also appear as such Choquet integrals, see [8,12]. The following graph illustrates the one-to-one correspondence between extremal coefficient functionals and CRSMs, cf. [38].



The classical *LePage series* representation [22] asserts that a symmetric stable random vector equals in distribution the sum of i.i.d. integrable random vectors scaled by successive points of the unit intensity Poisson process on the positive half-line. Its variant for max-stable processes is derived in [7]. In Section 4, we derive such a representation of a general max-stable random sup-measure as the maximum of i.i.d. copies of a random sup-measure scaled by successive Poisson points. The difficulty lies in the absence of a norm and a reference sphere in the space of (locally