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Importance sampling and statistical Romberg method for Lévy processes

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Abstract

An important family of stochastic processes arising in many areas of applied probability is the class of Lévy processes. Generally, such processes are not simulatable especially for those with infinite activity. In practice, it is common to approximate them by truncating the jumps at some cut-off size ε ($\varepsilon \searrow 0$). This procedure leads us to consider a simulatable compound Poisson process. This paper first introduces, for this setting, the statistical Romberg method to improve the complexity of the classical Monte Carlo method. Roughly speaking, we use many sample paths with a coarse cut-off ε^β, $\beta \in (0, 1)$, and few additional sample paths with a fine cut-off ε. Central limit theorems of Lindeberg–Feller type for both Monte Carlo and statistical Romberg method for the inferred errors depending on the parameter ε are proved with explicit formulas for the limit variances. This leads to an accurate description of the optimal choice of parameters. Afterwards, the authors propose a stochastic approximation method in order to find the optimal measure change by Esscher transform for Lévy processes with Monte Carlo and statistical Romberg importance sampling variance reduction. Furthermore, we develop new adaptive Monte Carlo and statistical Romberg algorithms and prove the associated central limit theorems. Finally, numerical simulations are processed to illustrate the efficiency of the adaptive statistical Romberg method that reduces at the same time the variance

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and the computational effort associated to the effective computation of option prices when the underlying asset process follows an exponential pure jump CGMY model.

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1. Introduction

Lévy processes arise in many areas of applied probability and specially in mathematical finance, where they become very fashionable since they can describe the observed reality of financial markets in a more accurate way than models based on Brownian motion (see e.g. Cont and Tankov [10] and Schoutens [32]). In particular in the pricing of financial securities we are interested in the computation of the real quantity $\mathbb{E}F(L_T)$, $T > 0$, where $(L_t)_{0\le t\le T}$ is a $\mathbb{R}^d$-valued pure jump Lévy process, $d \ge 1$ and $F : \mathbb{R}^d \to \mathbb{R}$ is a given function. In the literature, the computation of this quantity may be carried out using three different methods: Fourier transform methods, numerical methods for partial integro-differential equations and Monte Carlo methods. It is well known that the two first methods cannot cope with high dimensional problems. This gives a competitive edge for Monte Carlo methods in this setting. Therefore, the focus of this work is to study improved Monte Carlo methods using the statistical Romberg algorithm and the importance sampling technique. The statistical Romberg method is known for reducing the time complexity and the importance sampling technique is aimed at reducing the variance.

The Monte Carlo method consists of two steps. In the first step, we approximate the Lévy process $(L_t)_{0\le t\le T}$ by a simulatable Lévy process $(L_t^\varepsilon)_{0\le t\le T}$ with $\varepsilon > 0$. If ν denotes the Lévy measure of the Lévy process under consideration, then it is common to take $(L_t^\varepsilon)_{0\le t\le T}$ with Lévy measure $\nu_{|\{|x|\ge\varepsilon\}}$ and $\varepsilon \searrow 0$. This approximation is nothing but a compound Poisson process. In the second step, we approximate $\mathbb{E}\,F\left(L_T^\varepsilon\right)$ by $\frac{1}{N}\sum_{i=1}^N F(L_{T,i}^\varepsilon)$, where $(L_{T,i}^\varepsilon)_{1\le i\le N}$ is a sample of N independent copies of L_T^ε. Therefore, this Monte Carlo method (MC) is affected respectively by an approximation error and a statistical error:

$$\mathcal{E}_1(\varepsilon) := \mathbb{E}\left(F(L_T^\varepsilon) - F(L_T)\right) \quad \text{and} \quad \mathcal{E}_2(N) := \frac{1}{N}\sum_{i=1}^N F(L_{T,i}^\varepsilon) - \mathbb{E}F(L_T^\varepsilon).$$

On one hand, for a Lipschitz function F we have $\mathcal{E}_1(\varepsilon) = O(\sigma(\varepsilon))$, where $\sigma^2(\varepsilon) = \mathbb{E}|L_1 - L_1^\varepsilon|^2$ (see relation (6) for more details). On the other hand, the statistical error is controlled by the central limit theorem with order $1/\sqrt{N}$. Hence, optimizing the choice of the sample size N in the Monte Carlo method leads to $N = O(\sigma^{-2}(\varepsilon))$. Moreover, if we choose $N = \sigma^{-2}(\varepsilon)$ we prove a central limit theorem of Lindeberg–Feller type (see Theorem 3.1). Therefore, if we denote by $\mathcal{K}(\varepsilon)$ the cost of a single simulation of L_T^ε, then the mean total cost necessary to achieve the precision $\sigma(\varepsilon)$ is given by $C_{MC} = O(\mathcal{K}(\varepsilon)\sigma^{-2}(\varepsilon))$ (see Section 3.3).

In order to improve the performance of this method we use the idea of the statistical Romberg method introduced by Kebaier [22] in the setting of Euler Monte Carlo methods for stochastic differential equations driven by a standard Brownian Motion which is also related to the well known Romberg's method introduced by Talay and Tubaro in [33]. Inspired by this technique,