Available online at www.sciencedirect.com

ScienceDirect

Stochastic Processes and their Applications 126 (2016) 2038–2061

stochastic processes and their applications

www.elsevier.com/locate/spa

Perpetual American options in diffusion-type models with running maxima and drawdowns

Pavel V. Gapeev[a,*], Neofytos Rodosthenous[b]

[a] *London School of Economics, Department of Mathematics, Houghton Street, London WC2A 2AE, United Kingdom*
[b] *Queen Mary University of London, School of Mathematical Sciences, Mile End Road, London E1 4NS, United Kingdom*

Received 28 November 2013; received in revised form 28 April 2015; accepted 11 January 2016
Available online 9 February 2016

Abstract

We study perpetual American option pricing problems in an extension of the Black–Merton–Scholes model in which the dividend and volatility rates of the underlying risky asset depend on the running values of its maximum and maximum drawdown. The optimal exercise times are shown to be the first times at which the underlying asset hits certain boundaries depending on the running values of the associated maximum and maximum drawdown processes. We obtain closed-form solutions to the equivalent free-boundary problems for the value functions with smooth fit at the optimal stopping boundaries and normal reflection at the edges of the state space of the resulting three-dimensional Markov process. The optimal exercise boundaries for the perpetual American options on the maximum of the market depth with fixed and floating strikes are determined as the minimal solutions of certain first-order nonlinear ordinary differential equations.

MSC: primary 60G40; 34K10; 91G20; secondary 60J60; 34L30; 91B25

Keywords: Multi-dimensional optimal stopping problem; Brownian motion; Running maximum and running maximum drawdown process; Free-boundary problem; Instantaneous stopping and smooth fit; Normal reflection; A change-of-variable formula with local time on surfaces; Perpetual American options

* Corresponding author.
E-mail addresses: p.v.gapeev@lse.ac.uk (P.V. Gapeev), n.rodosthenous@qmul.ac.uk (N. Rodosthenous).

http://dx.doi.org/10.1016/j.spa.2016.01.003

1. Introduction

The main aim of this paper is to present closed-form solutions to the discounted optimal stopping problem of (2.4) for the running maximum S and the running maximum drawdown Y associated with the continuous process X defined in (2.1)–(2.2). This problem is related to the option pricing theory in mathematical finance, where the process X can describe the price of a risky asset (e.g. a stock) on a financial market. The value of (2.4) can therefore be interpreted as the rational (or no-arbitrage) price of a perpetual American option in a diffusion-type extension of the Black–Merton–Scholes model (see, e.g. Shiryaev [37, Chapter VIII; Section 2a], Peskir and Shiryaev [33, Chapter VII; Section 25], and Detemple [5], for an extensive overview of other related results in the area).

Optimal stopping problems for running maxima of some diffusion processes given linear costs were studied by Jacka [19], Dubins, Shepp, and Shiryaev [7], and Graversen and Peskir [14,15] among others, with the aim of determining the best constants in the corresponding maximal inequalities. A complete solution of a general version of the same problem was obtained in Peskir [29], by means of the established maximality principle, which is equivalent to the superharmonic characterization of the value function. Discounted optimal stopping problems for certain payoff functions depending on the running maxima of geometric Brownian motions were initiated by Shepp and Shiryaev [35,36] and then considered by Pedersen [28] and Guo and Shepp [16] among others, with the aim of computing rational values of perpetual American lookback (Russian) options. More recently, Guo and Zervos [17] derived solutions for discounted optimal stopping problems related to the pricing of perpetual American options with certain payoff functions depending on the running values of both the initial diffusion process and its associated maximum. Glover, Hulley, and Peskir [13] provided solutions to optimal stopping problems for integrals of functions depending on the running values of both the initial diffusion process and its associated minimum. The main feature of the resulting optimal stopping problems is that the normal-reflection condition holds for the value function at the diagonal of the state space of the two-dimensional continuous Markov process having the initial process and the running extremum as its components, which implies the characterization of the optimal boundaries as the extremal solutions of one-dimensional first-order nonlinear ordinary differential equations.

Asmussen, Avram, and Pistorius [1] considered perpetual American options with payoffs depending on the running maximum of some Lévy processes with two-sided jumps having phase-type distributions in both directions. Avram, Kyprianou, and Pistorius [2] studied exit problems for spectrally negative Lévy processes and applied the results to solving optimal stopping problems for payoff functions depending on the running values of the initial processes or their associated maxima. Optimal stopping games with payoff functions of such type were considered by Baurdoux and Kyprianou [3] and Baurdoux, Kyprianou, and Pardo [4] within the framework of models based on spectrally negative Lévy processes. Other complicated optimal stopping problems for the running maxima were considered in [11] for a jump-diffusion model with compound Poisson processes with exponentially distributed jumps and by Ott [27] and Kyprianou and Ott [21] (see also Ott [26]) for a model based on spectrally negative Lévy processes. More recently, Peskir [31,32] studied optimal stopping problems for three-dimensional Markov processes having the initial diffusion process as well as its maximum and minimum as the state space components. It was shown that the optimal boundary surfaces depending on the maximum and minimum of the initial process provide the maximal and minimal solutions of the associated systems of first-order non-linear partial differential equations.