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Stochastic Processes and their Applications 126 (2016) 2123–2162

stochastic processes and their applications

www.elsevier.com/locate/spa

Simulation of BSDEs with jumps by Wiener Chaos expansion

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Received 17 February 2015; received in revised form 20 January 2016; accepted 21 January 2016
Available online 1 February 2016

Abstract

We present an algorithm to solve BSDEs with jumps based on Wiener Chaos Expansion and Picard's iterations. This paper extends the results given in Briand and Labart (2014) to the case of BSDEs with jumps. We get a forward scheme where the conditional expectations are easily computed thanks to chaos decomposition formulas. Concerning the error, we derive explicit bounds with respect to the number of chaos, the discretization time step and the number of Monte Carlo simulations. We also present numerical experiments. We obtain very encouraging results in terms of speed and accuracy.

MSC: 60H10; 60J75; 60H35; 65C05; 65G99; 60H07

Keywords: Backward stochastic differential equations with jumps; Wiener Chaos expansion; Numerical method

1. Introduction

In this paper we are interested in the numerical approximation of solutions (Y, Z, U) to backward stochastic differential equations (BSDEs in the sequel) with jumps of the following form

$$Y_t = \xi + \int_t^T f(s, Y_s, Z_s, U_s)\, ds - \int_t^T Z_s dB_s - \int_{]t,T]} U_s d\tilde{N}_s, \quad 0 \le t \le T, \tag{1}$$

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http://dx.doi.org/10.1016/j.spa.2016.01.006
0304-4149/

where B is a 1-dimensional standard Brownian motion and $\tilde{N}$ is a compensated Poisson process independent from B, i.e. $\tilde{N}_t := N_t - \kappa t$ and $\{N_t\}_{t\geq 0}$ is a Poisson process with intensity $\kappa > 0$. The terminal condition ξ is a real-valued $\mathcal{F}_T$-measurable random variable where $\{\mathcal{F}_t\}_{0\leq t\leq T}$ stands for the augmented natural filtration associated with B and N. Under standard Lipschitz assumptions on the driver f, the existence and uniqueness of the solution have been stated by Tang and Li [23], generalizing the seminal paper of Pardoux and Peng [18].

The main objective of this paper is to propose a numerical method to approximate the solution (Y, Z, U) of (1). In the no-jump case, there exist several methods to simulate (Y, Z). The most popular one is the method based on the dynamic programming equation, introduced by Briand, Delyon and Mémin [6]. In the Markovian case, the rate of convergence of the method has been studied by Zhang [24] and Bouchard and Touzi [4]. From a numerical point of view, the main difficulty in solving BSDEs is to compute conditional expectations. Different approaches have been proposed: Malliavin calculus [4], regression methods [10] and quantization techniques [2]. In the general case (i.e. for a terminal condition which is not necessarily Markovian), Briand and Labart [7] have proposed a forward scheme based on Wiener chaos expansion and Picard's iterations. Thanks to the chaos decomposition formulas, conditional expectations are easily computed, which leads to an efficient, fully implementable scheme. In case of BSDEs driven by a Poisson random measure, Bouchard and Elie [3] have proposed a scheme based on the dynamic programming equation and studied the rate of convergence of the method when the terminal condition is given by $\xi = g(X_T)$, where g is a Lipschitz function and X is a forward process. More recently, Geiss and Steinicke [9] have extended this result to the case of a terminal condition which may be a Borel function of finitely many increments of the Lévy forward process X which is not necessarily Lipschitz but only satisfies a fractional smoothness condition. In the case of jumps driven by a compensated Poisson process, Lejay, Mordecki and Torres [15] have developed a fully implementable scheme based on a random binomial tree, following the approach proposed by Briand, Delyon and Mémin [5].

In this paper, we extend the algorithm based on Picard's iterations and Wiener chaos expansion introduced in [7] to the case of BSDEs with jumps. Our starting point is the use of Picard's iterations: $(Y^0, Z^0, U^0) = (0, 0, 0)$ and for $q \in \mathbb{N}$,

$$Y_t^{q+1} = \xi + \int_t^T f\left(s, Y_s^q, Z_s^q, U_s^q\right) ds - \int_t^T Z_s^{q+1} \cdot dB_s - \int_{]t,T]} U_s^{q+1} d\tilde{N}_s,$$
$$0 \leq t \leq T.$$

Writing this Picard scheme in a forward way gives

$$Y_t^{q+1} = \mathbb{E}\left(\xi + \int_0^T f\left(s, Y_s^q, Z_s^q, U_s^q\right) ds \Big| \mathcal{F}_t\right) - \int_0^t f\left(s, Y_s^q, Z_s^q, U_s^q\right) ds,$$
$$Z_t^{q+1} = \mathbb{E}\left(D_t^{(0)} Y_t^{q+1} \Big| \mathcal{F}_{t^-}\right) = \mathbb{E}\left(D_t^{(0)}\left(\xi + \int_0^T f\left(s, Y_s^q, Z_s^q, U_s^q\right) ds\right) \Big| \mathcal{F}_{t^-}\right),$$
$$U_t^{q+1} = \mathbb{E}\left(D_t^{(1)} Y_t^{q+1} \Big| \mathcal{F}_{t^-}\right) = \mathbb{E}\left(D_t^{(1)}\left(\xi + \int_0^T f\left(s, Y_s^q, Z_s^q, U_s^q\right) ds\right) \Big| \mathcal{F}_{t^-}\right),$$

where $D_t^{(0)} X$ (resp. $D_t^{(1)} X$) stands for the Malliavin derivative of the random variable X with respect to the Brownian motion (resp. w.r.t. the Poisson process).