Available online at www.sciencedirect.com

ScienceDirect

Stochastic Processes and their Applications 125 (2015) 2700–2726

stochastic processes and their applications

www.elsevier.com/locate/spa

On the limiting spectral distribution for a large class of symmetric random matrices with correlated entries

Marwa Banna[a], Florence Merlevède[a,*], Magda Peligrad[b]

[a] *Université Paris Est, LAMA (UMR 8050), UPEMLV, CNRS, UPEC, 5 Boulevard Descartes, 77 454 Marne La Vallée, France*

[b] *Department of Mathematical Sciences, University of Cincinnati, PO Box 210025, Cincinnati, OH 45221-0025, USA*

Received 23 September 2013; received in revised form 7 November 2014; accepted 27 January 2015
Available online 9 February 2015

Abstract

For symmetric random matrices with correlated entries, which are functions of independent random variables, we show that the asymptotic behavior of the empirical eigenvalue distribution can be obtained by analyzing a Gaussian matrix with the same covariance structure. This class contains both cases of short and long range dependent random fields. The technique is based on a blend of blocking procedure and Lindeberg's method. This method leads to a variety of interesting asymptotic results for matrices with dependent entries, including applications to linear processes as well as nonlinear Volterra-type processes entries.

MSC: 60F15; 60G60; 60G10; 62E20

Keywords: Random matrices; Correlated entries; Sample covariance matrices; Weak dependence; Limiting spectral distribution

1. Introduction

The limiting spectral distribution for symmetric matrices with correlated entries received a lot of attention in the last two decades. The starting point is deep results for symmetric matrices with

* Corresponding author.

E-mail addresses: marwa.banna@u-pem.fr (M. Banna), florence.merlevede@u-pem.fr (F. Merlevède), peligrm@ucmail.uc.edu (M. Peligrad).

http://dx.doi.org/10.1016/j.spa.2015.01.010
0304-4149/

correlated Gaussian entries by Khorunzhy and Pastur [13], Boutet de Monvel et al. [6], Boutet de Monvel and Khorunzhy [5], Chakrabarty et al. [7] among others. On the other hand there is a sustained effort for studying linear filters of independent random variables as entries of a matrix. For instance, Anderson and Zeitouni [1] considered symmetric matrices with entries that are linear processes of finite range of independent random variables. Hachem et al. [12] considered large sample covariance matrices whose entries are modeled by a short memory linear process of infinite range with independent Gaussian innovations. Bai and Zhou [3], Yao [18], Banna and Merlevède [4] and Merlevède and Peligrad [14], among others, treated large covariance matrices based on an independent structure between columns and correlated random variables in rows.

In this paper we consider symmetric random matrices whose entries are functions of independent and identically distributed (i.i.d.) real-valued random variables. Such kind of processes provide a very general framework for stationary ergodic random fields. Our main goal is to reduce the study of the limiting spectral distribution to the same problem for a Gaussian matrix having the same covariance structure as the underlying process. In this way we prove the universality and we are able to formulate various limiting results for large classes of matrices. We also treat large sample covariance matrices with correlated entries, known under the name of Gram matrices. Our proofs are based on the large-small block arguments, a method which, in one dimensional setting, is going back to Bernstein. Then, we apply a variant of the so-called Lindeberg method, namely we develop a block Lindeberg method, where we replace at one time a big block of random variables with a Gaussian one with the same covariance structure. Lindeberg method is popular with these type of problems. Replacing only one variable at one time with a Gaussian one, Chatterjee [8] treated random matrices with exchangeable entries.

Our paper is organized in the following way. Section 2 contains the main results for symmetric random matrices and sample covariance matrices. As an intermediate step we also treat matrices based on K-dependent random fields, results that have interest in themselves (see Theorem 11 in Section 4.1). In Section 3, we give applications to matrices with entries which are either linear random fields or nonlinear random fields as Volterra-type processes. The main proofs are included in Section 4. In Section 5 we prove a concentration of spectral measure inequality for a row-wise K-dependent random matrix and we also mention some of the technical results used in the paper.

Here are some notations used all along the paper. The notation $[x]$ is used to denote the integer part of any real x. For any positive integers a, b, the notation $\mathbf{0}_{a,b}$ means a matrix with 0 entries of size $a \times b$, whereas the notation $\mathbf{0}_a$ means a row vector of size a. For a matrix A, we denote by A^T its transpose matrix and by $\operatorname{Tr}(A)$ its trace. We shall use the notation $\|X\|_r$ for the $\mathbb{L}^r$-norm ($r \geq 1$) of a real valued random variable X.

For any square matrix A of order n with only real eigenvalues $\lambda_1 \leq \cdots \leq \lambda_n$, its spectral empirical measure and its spectral distribution function are respectively defined by

$$\nu_A = \frac{1}{n}\sum_{i=1}^{n} \delta_{\lambda_i} \quad \text{and} \quad F_n^A(x) = \frac{1}{n}\sum_{k=1}^{n} \mathbf{1}_{\{\lambda_k \leq x\}}.$$

The Stieltjes transform of F^A is given by

$$S_A(z) = \int \frac{1}{x-z} dF^A(x) = \frac{1}{n}\operatorname{Tr}(A - z\mathbf{I}_n)^{-1},$$

where $z = u + \mathrm{i}v \in \mathbb{C}^+$ (the set of complex numbers with positive imaginary part), and $\mathbf{I}_n$ is the identity matrix of order n.