Available online at www.sciencedirect.com

ScienceDirect

Stochastic Processes and their Applications 126 (2016) 680–702

stochastic processes and their applications

www.elsevier.com/locate/spa

Quantitative results for the Fleming–Viot particle system and quasi-stationary distributions in discrete space

Bertrand Cloez[a,b,*], Marie-Noémie Thai[c]

[a] *UMR INRA-SupAgro MISTEA, Montpellier, France*
[b] *EPI INRA-INRIA MODEMIC, Sophia-Antipolis, France*
[c] *Laboratoire d'Analyse et de Mathématiques Appliquées, CNRS UMR8050, Université Paris-Est Marne-la-Vallée, France*

Received 1 August 2014; received in revised form 24 September 2015; accepted 25 September 2015
Available online 9 October 2015

Abstract

We show, for a class of discrete Fleming–Viot (or Moran) type particle systems, that the convergence to the equilibrium is exponential for a suitable Wasserstein coupling distance. The approach provides an explicit quantitative estimate on the rate of convergence. As a consequence, we show that the conditioned process converges exponentially fast to a unique quasi-stationary distribution. Moreover, by estimating the two-particle correlations, we prove that the Fleming–Viot process converges, uniformly in time, to the conditioned process with an explicit rate of convergence. We illustrate our results on the examples of the complete graph and of N particles jumping on two points.

MSC: 60K35; 60B10; 37A25

Keywords: Fleming–Viot process; quasi-stationary distributions; Coupling; Wasserstein distance; Chaos propagation; Commutation relation

* Corresponding author at: UMR INRA-SupAgro MISTEA, Montpellier, France.
E-mail addresses: bertrand.cloez@supagro.inra.fr (B. Cloez), marie-noemie.thai@univ-paris-est.fr (M.-N. Thai).

http://dx.doi.org/10.1016/j.spa.2015.09.016
0304-4149/

1. Introduction

This paper deals with a (time-continuous) Moran type model, referred to as the Fleming–Viot process in the literature [5,18], which approximates Markov semigroup conditioned on non-absorption. Briefly, when considering a time-continuous Markov chain, an interesting question is about the quasi-stationary distribution of the process which is killed at some rate, see for instance [10,25]. Instead of conditioning on non-killing, it is possible to start N copies of the Markov chain and, instead of being killed, one chain jumps randomly on the state of another one. The resulting process is a version of the Moran model that we will call Fleming–Viot. While the convergence of the large-population limit of the Moran model to the quasi-stationary distribution was already shown under some assumptions [13,18,31], the present paper is concerned with deriving bounds for the rate of convergence. Our first main result, namely Theorem 1.1, establishes the exponential ergodicity of the particle system with an explicit rate. This seems to be a novelty. As a consequence, we prove that the correlations between particles vanish uniformly in time, see Theorems 1.3 and 2.6. This is also a new result even if [18] gives a similar bound heavily depending on time. As application, we also give new proofs for some more classical but important results as a rate of convergence as N tends to infinity (Theorem 1.2) which can be compared to the results of [13,20,31], a quantitative convergence of the conditioned semi-group (Corollary 1.4) comparable to the results of [14,24] and uniform bound (in time) as N tends to infinity (see Corollary 1.5), which seems to be new in discrete space but already proven for diffusion processes in [28] with an approach based on martingale inequality and spectral theory associated to Schrödinger equation.

Let us now be more precise and introduce our model. Let $(Q_{i,j})_{i,j\in F^*}$ be the transition rate matrix of an irreducible and positive recurrent continuous time Markov process on a discrete and countable state space F^*. Set $F = F^* \cup \{0\}$ where $0 \notin F^*$ and let $p_0 : F^* \mapsto \mathbb{R}_+$ be a non-null function. The generator of the Markov process $(X_t)_{t\geq 0}$, with transition rate Q and death rate p_0, when applied to bounded functions $f : F \mapsto \mathbb{R}$, reads

$$Gf(i) = p_0(i)(f(0) - f(i)) + \sum_{j\in F^*} Q_{i,j}(f(j) - f(i)),$$

for every $i \in F^*$ and $Gf(0) = 0$. If this process does not start from 0, then it moves according to the transition rate Q until it jumps to 0 with rate p_0; the state 0 is absorbing. Consider the process $(X_t)_{t\geq 0}$ generated by G with initial law μ and denote by μT_t its law at time t conditioned on non absorption up to time t. That is defined, for all non-negative functions f on F^*, by

$$\mu T_t f = \frac{\mu P_t f}{\mu P_t \mathbf{1}_{\{0\}^c}} = \frac{\sum_{y\in F^*} P_t f(y)\mu(y)}{\sum_{y\in F^*} P_t \mathbf{1}_{\{0\}^c}(y)\mu(y)},$$

where $(P_t)_{t\geq 0}$ is the semigroup generated by G and we use the convention $f(0) = 0$. For every $x \in F^*, k \in F^*$ and non-negative function f on F^*, we also set

$$T_t f(x) = \delta_x T_t f \quad \text{and} \quad \mu T_t(k) = \mu T_t \mathbf{1}_{\{k\}}, \quad \forall t \geq 0.$$

A quasi-stationary distribution (QSD) for G is a probability measure $\nu_{\rm qs}$ on F^* satisfying, for every $t \geq 0, \nu_{\rm qs} T_t = \nu_{\rm qs}$. The QSD are neither well understood, nor easily amenable to simulation. To avoid these difficulties, Burdzy, Hołyst, Ingerman, March [5], and Del Moral, Guionnet, Miclo [12,13] introduced, independently from each other, a Fleming–Viot or Moran