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Stochastic Processes and their Applications 126 (2016) 832–856

stochastic processes and their applications

www.elsevier.com/locate/spa

Stable random fields, point processes and large deviations

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Received 9 December 2014; received in revised form 11 August 2015; accepted 29 September 2015
Available online 21 October 2015

Abstract

We investigate the large deviation behaviour of a point process sequence based on a stationary symmetric α-stable $(0 < \alpha < 2)$ discrete-parameter random field using the framework of Hult and Samorodnitsky (2010). Depending on the ergodic theoretic and group theoretic structures of the underlying nonsingular group action, we observe different large deviation behaviours of this point process sequence. We use our results to study the large deviations of various functionals (e.g., partial sum, maxima, etc.) of stationary symmetric stable fields.

MSC: primary 60F10; 60G55; 60G52; secondary 60G60; 60G70; 37A40

Keywords: Large deviations; Point processes; Stable processes; Random fields; Extreme value theory; Nonsingular group actions

1. Introduction

In this paper, we investigate the large deviation behaviours of point processes and partial sums of stationary *symmetric α-stable* ($S\alpha S$) random fields with $\alpha \in (0, 2)$. A random field $\mathbf{X} := \{X_t\}_{t\in\mathbb{Z}^d}$ is called a *stationary symmetric α-stable* discrete-parameter random field if for

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http://dx.doi.org/10.1016/j.spa.2015.09.020

all $k \geq 1$, for all $s, t_1, t_2, \ldots, t_k \in \mathbb{Z}^d$, and for all $c_1, c_2, \ldots, c_k \in \mathbb{R}$, $\sum_{i=1}^{k} c_i X_{t_i+s}$ follows an $S\alpha S$ distribution that does not depend on s. See, for example, [38] for detailed descriptions on $S\alpha S$ distributions and processes.

The study of rare events and large deviations for heavy-tailed distributions and processes has been of considerable importance starting from the classical works of Heyde [11–13], Nagaev [23,24], Nagaev [25]; see also the technical report of Cline and Hsing [5]. Some of the more recent works in this area include Mikosch and Samorodnitsky [22], Rachev and Samorodnitsky [26], Hult et al. [16], Denisov et al. [10], Hult and Samorodnitsky [17], etc. When studying the probability of rare events, it is usually important not only to determine the size and the frequency of clusters of extreme values but also to capture the intricate structure of the clusters. For this reason, Hult and Samorodnitsky [17] developed a theory to study large deviation behaviours at the level of point processes to get a better grasp on how rare events occur. Their work relies on convergence of measures that was introduced in [15]. See also the recent works of Das et al. [7,21], which extended this convergence to more general situations.

Inspired by the works of Davis and Resnick [9] and Davis and Hsing [8], Resnick and Samorodnitsky [29] studied the asymptotic behaviour of a point process sequence induced by a stationary symmetric stable process. This work was extended to stable random fields by Roy [33]. In the present work, we take a marked and slightly stronger version of the point process sequence considered in [33]. More precisely, we keep track of the random field when the index parameter lies in a neighbourhood and use the framework introduced by Hult and Samorodnitsky [17] to investigate the corresponding large deviation behaviour. The advantage of this setup is that we get to know the order in which large values arrive. This is important, for example, in the study of ruin probabilities and long strange segments; see [16,17]. In our case, we observe that the point process large deviation principle depends on the ergodic theoretic and group theoretic properties of the underlying nonsingular $\mathbb{Z}^d$-action through the works of Rosiński [30,31] and Roy and Samorodnitsky [35]. Just as in [36,37] (see also [34]), we notice a phase transition that can be regarded as a passage from shorter to longer memory.

The paper is organized as follows. In Section 2, we present background on ergodic theory of nonsingular group actions and integral representations of $S\alpha S$ random fields, and describe a special type of convergence of measures. The large deviation behaviours of the associated point processes are considered separately for stationary $S\alpha S$ random fields generated by dissipative group actions (reflecting shorter memory) in Section 3, and generated by conservative group actions (reflecting longer memory) in Section 4. Finally, in Section 5, we obtain the large deviation principle for the partial sum sequence of a stationary $S\alpha S$ random field using a continuous mapping theorem.

We introduce some notations that we are going to use throughout this paper. For two sequences of real numbers $\{a_n\}_{n\in\mathbb{N}}$ and $\{b_n\}_{n\in\mathbb{N}}$ the notation $a_n \sim b_n$ means $a_n/b_n \to 1$ as $n \to \infty$. For $u, v \in \mathbb{Z}^d$, $u = (u_1, u_2, \ldots, u_d) \leq v = (v_1, v_2, \ldots, v_d)$ means $u_i \leq v_i$ for all $i = 1, 2, \ldots, d$; $[u, v]$ is the set $\{t \in \mathbb{Z}^d : u \leq t \leq v\}$; $\|u\|_\infty := \max_{1\leq i\leq d} |u_i|$ and $\mathbf{0}_d = (0, 0, \ldots, 0)$, $\mathbf{1}_d = (1, 1, \ldots, 1)$ are elements of $\mathbb{Z}^d$. For $x \in \mathbb{R}$, we define $x^+ := \max(x, 0)$ and $x^- := \max(-x, 0)$. Weak convergence is denoted by $\Rightarrow$. For some standard Borel space $(S, \mathcal{S})$ with σ-finite measure μ we define the space $L^\alpha(S, \mu) := \{f : S \to \mathbb{R} \text{ measurable} : \|f\|_\alpha < \infty\}$ with $\|f\|_\alpha := \left(\int_S |f(s)|^\alpha \, \mu(ds)\right)^{1/\alpha}$. For two random variables Y, Z (not necessarily defined on the same probability space), we write $Y \stackrel{d}{=} Z$ if Y and Z are identically distributed. For two random fields $\{Y_t\}_{t\in\mathbb{Z}^d}$ and $\{Z_t\}_{t\in\mathbb{Z}^d}$, the notation $Y_t \stackrel{d}{=} Z_t,\ t \in \mathbb{Z}^d$ means that they have same finite-dimensional distributions.