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Stochastic Processes and their Applications 126 (2016) 906–929

stochastic processes and their applications

www.elsevier.com/locate/spa

Random locations, ordered random sets and stationarity

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Received 15 December 2014; received in revised form 7 July 2015; accepted 7 October 2015
Available online 23 October 2015

Abstract

Intrinsic location functional is a large class of random locations closely related to stationary processes. In this paper the author firstly identifies a subclass of intrinsic location functional and proves that it characterizes stationary increment processes. Then a generalization of intrinsic location functional is introduced and its relationship with intrinsic location functional is discussed. Finally we develop representation results using ordered random sets and piecewise linear functions. It is proved that each random location corresponds to the maximal element in a random set according to certain order, and the locations change in a specific way when the path is translated.
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MSC: primary 60G10; 60G17

Keywords: Random locations; Stationary increment processes; Ordered random sets

1. Introduction

Stationarity has been an essential concept in stochastic processes since very long, both due to its theoretical importance and to its extensive use in modeling. Many related problems, especially extreme values of stationary processes, have attracted intensive and ongoing research interests. The classical text [1] and the new book [2] are both excellent sources for summaries of existing results and literature reviews. Meanwhile, the random locations of stationary processes, such as the location of the path supremum over an interval or the first hitting time of certain level over an

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http://dx.doi.org/10.1016/j.spa.2015.10.004
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interval, have received relatively less attention, particularly in a general setting, when the process is not from one of the few well studied "nice" classes.

In the paper [3], the authors introduced a new notion called "intrinsic location functional", as an abstraction of the common random locations often considered. More precisely, let H be a space of real valued functions on $\mathbb{R}$, closed under shift. That is, for any $f \in H$ and $c \in \mathbb{R}$, the function $\theta_c f$, defined by $\theta_c f(x) = f(x+c)$, $x \in \mathbb{R}$ is also in H. Examples of H include the space of all continuous functions $\mathcal{C}(\mathbb{R})$, the space of all càdlàg functions $\mathcal{D}(\mathbb{R})$, the space of all upper semi-continuous functions, *etc*. Equip H with the cylindrical σ-field. Let $\mathcal{I}$ be the set of all compact, non-degenerate intervals in $\mathbb{R}$: $\mathcal{I} = \{[a,b] : a < b, [a,b] \subset \mathbb{R}\}$.

Definition 1.1. A mapping $L : H \times \mathcal{I} \to \mathbb{R} \cup \{\infty\}$ is called an intrinsic location functional, if it satisfies the following conditions.

1. For every $I \in \mathcal{I}$ the map $L(\cdot, I) : H \to \mathbb{R} \cup \{\infty\}$ is measurable.
2. For every $f \in H$ and $I \in \mathcal{I}$, $L(f, I) \in I \cup \{\infty\}$.
3. (Shift compatibility) For every $f \in H$, $I \in \mathcal{I}$ and $c \in \mathbb{R}$,
$$L(f, I) = L(\theta_c f, I - c) + c,$$
where $I - c$ is the interval I shifted by $-c$, and $\infty + c = \infty$.
4. (Stability under restrictions) For every $f \in H$ and $I_1, I_2 \in \mathcal{I}$, $I_2 \subseteq I_1$,
$$\text{if } L(f, I_1) \in I_2, \quad \text{then } L(f, I_2) = L(f, I_1).$$
5. (Consistency of existence) For every $f \in H$ and $I_1, I_2 \in \mathcal{I}$, $I_2 \subseteq I_1$,
$$\text{if } L(f, I_2) \neq \infty, \quad \text{then } L(f, I_1) \neq \infty.$$

It is not difficult to realize that intrinsic location functional is an abstraction of common random locations such as the location of the path supremum/infimum over an interval, the first/last hitting time over an interval, among many others. Interested readers are invited to see [3] for more examples and counterexamples of intrinsic location functionals. Notice that in the definition we included ∞ as a possible value. This corresponds to the fact that not all the random locations are necessarily well-defined for all the paths. For instance, a path can lie above certain level over the whole interval of interest, leaving the first/last hitting time undefined. Here and later, we always assign ∞ as the value of an intrinsic location functional when it is otherwise undefined. Accordingly, the σ-field used for $\mathbb{R} \cup \{\infty\}$ is generated by the Borel σ-field plus ∞ as a singleton.

It turns out that, despite the huge variety of the origins and natures of these random locations, the common points that they share, now summarized in the definition of intrinsic location functional, are sufficient to guarantee many interesting and important properties of their distributions for stationary processes. The majority of these properties are firstly studied in [4] and [5], for the location of path supremum over compact intervals.

Fix a path space H. Let us denote the stochastic process by $\mathbf{X}$, with all sample paths in H, and the intrinsic location functional by L. Then for each fixed interval $I = [a,b] \in \mathcal{I}$, $L(\mathbf{X}, I)$ is a random variable taking value on $\mathbb{R} \cup \{\infty\}$. Denote its cumulative distribution function by $F_{\mathbf{X},I}$ or $F_{\mathbf{X},[a,b]}$. When the stationarity is assumed, it is clear that the location of the interval I will not affect the distribution of $L(\mathbf{X}, I) - a$, as long as the length of the interval, $|I| = b - a$, remains constant. In this case we often fix the starting point a to be 0, and use the shorter notation $F_{\mathbf{X},b}$.

Theorem 1.2 ([3]). *Let L be an intrinsic location functional and $\mathbf{X} = (X(t), t \in \mathbb{R})$ a stationary process. Then the restriction of the law $F_{\mathbf{X},T}$ to the interior $(0, T)$ of the interval*

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