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Edwards–Wilkinson fluctuations in the Howitt–Warren flows

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Abstract

We study current fluctuations in a one-dimensional interacting particle system known as the dual smoothing process that is dual to random motions in a Howitt–Warren flow. The Howitt–Warren flow can be regarded as the transition kernels of a random motion in a continuous space–time random environment. It turns out that the current fluctuations of the dual smoothing process fall in the Edwards–Wilkinson universality class, where the fluctuations occur on the scale $t^{1/4}$ and the limit is a universal Gaussian process. Along the way, we prove a quenched invariance principle for a random motion in the Howitt–Warren flow. Meanwhile, the centered quenched mean process of the random motion also converges on the scale $t^{1/4}$, where the limit is another universal Gaussian process.

MSC: 60K35; 60K37; 60F17

Keywords: Edwards–Wilkinson fluctuations; Howitt–Warren flows; Sticky Brownian motions; Quenched invariance principle

1. Introduction

1.1. Overview

In the review article [17], Seppäläinen discussed the processes of particle currents in several dynamical stochastic systems of particles on the one-dimensional integer lattice. It turns out that

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for independent random walks, independent random walks in an i.i.d. space random environment, and the random average process (RAP), there is a universal limit for the current fluctuations on the scale $n^{1/4}$, which is a certain family of self-similar Gaussian processes. These three models all belong to the so-called Edwards–Wilkinson (EW) universality class. Two more recent examples in the EW class are one-dimensional Hammersley's harness process [18] and the Atlas model [6]. In the EW class the limiting current fluctuations are described by the linear stochastic heat equation $Z_t = \upsilon Z_{xx} + \dot{W}$ where $\dot{W}$ is space–time white noise and υ is a non-zero parameter. In contrast, asymmetric simple exclusion process and a class of totally asymmetric zero range processes have nontrivial current fluctuations on the scale $n^{1/3}$, and the Tracy–Widom distributions are the universal limits. These two models belong to the Kardar–Parisi–Zhang (KPZ) universality class. More discussions about EW and KPZ universality classes and their relations can be found in [17,5]. However, all the models that were shown to be in the EW universality up to now are discrete models defined on $\mathbb{Z}$. The motivation of this paper is to present a model in continuous space and time that also falls in the EW class.

Recently, in [13] Le Jan and Raimond introduced the so-called stochastic flow of kernels, which is a collection of random probability kernels. Heuristically, a stochastic flow of kernels can be interpreted as the transition kernels of a Markov process in a space–time random environment, where restrictions of the environment to disjoint space–time regions are independent and the law of the environment satisfies translation-invariance in space and time. Given the environment, one can sample n independent Markov processes (random motions) and then average over the environment. This leads to a Markov process known as the n-point motion of the flow and their joint law satisfies a natural consistency condition: the marginal distribution of any k components of an n-point motion is necessarily a k-point motion. The main result of Le Jan and Raimond [13] is that any family of Feller processes that is consistent in this way gives rise to a unique stochastic flow of kernels. Using martingale problems, Howitt and Warren later constructed a class of consistent Feller processes on $\mathbb{R}$ which are Brownian motions with sticky interaction when they meet. Thus by the fundamental result of Le Jan and Raimond, this class of Feller process determines the unique stochastic flow of kernels which is now called the Howitt–Warren flow. In [16], Schertzer, Sun and Swart showed that the Howitt–Warren flows can be realized as the transition kernels of a random motion in a space–time environment, constructed explicitly from the Brownian web and Brownian net. Thus the heuristic interpretation above naturally becomes rigorous.

Dual smoothing process dual to Howitt–Warren flows, which is a function-valued process, was also introduced in [16]. As a continuum space–time analogue of RAP, dual smoothing process can be thought of as the evolution of the interface height function in a growth model as well. In one dimension, conservative interacting particle systems can always be equivalently formulated as interface models. Here the connection goes by regarding the gradient of the interface height function as a measure governing the distribution of the particles. The movement of particle currents can then be viewed as deposition or removal of particles from the growing interface. With such an equivalent formulation, the current process maps directly to the height function. We will show that on the scale $t^{1/4}$, the fluctuations of the height function (dual smoothing process), which is the first continuum space–time model shown to be in the EW class, converges weakly to a universal Gaussian process. Along the way, we will show that for random motions in the Howitt–Warren flows, the process of the centered quenched means, indexed by space and time, converges to a Gaussian process after rescaling by $t^{-1/4}$. Moreover, we will prove a quenched invariance principle for random motion in the Howitt–Warren flows, which is of independent interest.