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Strong Markov property of determinantal processes with extended kernels

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Abstract

Noncolliding Brownian motion (Dyson's Brownian motion model with parameter $\beta = 2$) and noncolliding Bessel processes are determinantal processes; that is, their space–time correlation functions are represented by determinants. Under a proper scaling limit, such as the bulk, soft-edge and hard-edge scaling limits, these processes converge to determinantal processes describing systems with an infinite number of particles. The main purpose of this paper is to show the strong Markov property of these limit processes, which are determinantal processes with the extended sine kernel, extended Airy kernel and extended Bessel kernel, respectively. We also determine the quasi-regular Dirichlet forms and infinite-dimensional stochastic differential equations associated with the determinantal processes.

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1. Introduction

In a system of N independent one-dimensional diffusion processes, if we impose the condition that the particles never collide with one another, we obtain an interacting particle system with

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a long-range strong repulsive force between any pair of particles. We call such a system a noncolliding diffusion process. In 1962 Dyson [2] showed that when the individual diffusion process is one-dimensional Brownian motion (BM), it is related to a matrix-valued process. We call this stochastic process *the Dyson model*. The model solves the stochastic differential equation (SDE)

$$dX_j(t) = dB_j(t) + \sum_{\substack{k:k\neq j\\k=1}}^{N} \frac{dt}{X_j(t) - X_k(t)}, \quad j \in \mathbb{I}_N \equiv \{1, 2, \ldots, N\}, \tag{1.1}$$

where $B_j(t)$, $j = 1, 2, \ldots, N$ are independent one-dimensional BMs. When the individual diffusion process is a squared Bessel process with index $\nu > -1$, the noncolliding diffusion process is called a *noncolliding squared Bessel process*, satisfying the SDE

$$dZ_j^\nu(t) = 2\sqrt{Z_j^\nu(t)}dB_j(t) + 2(\nu+1)dt + \sum_{\substack{k:k\neq j\\k=1}}^{N} \frac{4Z_j^\nu(t)\,ds}{Z_j^\nu(t) - Z_k^\nu(t)}, \quad j \in \mathbb{I}_N \tag{1.2}$$

and if $-1 < \nu < 0$ having a reflection wall at the origin. These processes dynamically simulate the eigenvalue statistics of the Gaussian random matrix ensembles studied in random matrix theory [17,3].

Let $\mathfrak{M}$ be the space of nonnegative integer-valued Radon measures on $\mathbb{R}$. This space is a Polish space with the *vague topology*. The space can be regarded as a configuration space of unlabeled particles in $\mathbb{R}$. A probability measure on the space $\mathfrak{M}$ is called a *determinantal point process* (DPP) or *Fermion point process*, if its correlation functions are generally represented by determinants [30,31]. In this paper we say that an $\mathfrak{M}$-valued process $\Xi(t)$ is *determinantal* if the multitime correlation functions for any chosen series of times are represented by determinants. It has been shown that for any initial configuration $\xi^N = \sum_{j=1}^N \delta_{x_j}$, the unlabeled process $\Xi^N(t) = \sum_{j=1}^N \delta_{X_j(t)}$ is a determinantal process for the Dyson model in [10], and for the noncolliding Bessel process in [11].

Suppose that $(\Xi(t), \mathbb{P}^{\xi^N})$ is a noncolliding diffusion process starting from the initial configuration ξ^N of a finite number of particles. Let ξ be the configuration of an infinite number of particles, and $\xi_{[-L,L]}$ denote the restriction of ξ on the set $[-L, L]$. When the sequence of processes $(\Xi(t), \mathbb{P}^{\xi_{[-L,L]}})$, $L \in \mathbb{N} = \{1, 2, \ldots\}$, converges to an $\mathfrak{M}$-valued process starting from the configuration ξ, we denote the limit process by $(\Xi(t), \mathbb{P}^{\xi})$, and say that process $(\Xi(t), \mathbb{P}^{\xi})$ is well-defined. Sufficient conditions were given for configuration ξ that limit process is well defined, for noncolliding BM with bulk scaling [10], noncolliding BM with soft edge scaling [9], and noncolliding Bessel processes with hard-edge scaling [11]. We denote the associated limit processes by $(\Xi(t), \mathbb{P}^{\xi}_{\sin})$, $(\Xi(t), \mathbb{P}^{\xi}_{\mathrm{Ai}})$ and $(\Xi(t), \mathbb{P}^{\xi}_{\nu})$, respectively.

Let $\mu_{\sin}$, μ_{Ai} and μ_{J_ν} be the DPPs with the sine kernel (2.1), the Airy kernel (2.2) and the Bessel kernel (2.3), respectively. They are the probability measures obtained in the bulk scaling limit and soft-edge scaling limit of the eigenvalue distribution (3.10) in the *Gaussian unitary ensemble* (GUE), and in the hard-edge scaling limit of the eigenvalue distribution (3.15) in the *chiral Gaussian unitary ensemble* (chGUE), respectively. It was shown in [12] that processes $(\Xi(t), \mathbb{P}^{\xi}_{\sin})$, $(\Xi(t), \mathbb{P}^{\xi}_{\mathrm{Ai}})$ and $(\Xi(t), \mathbb{P}^{\xi}_{\nu})$ have DPPs $\mu_{\sin}$, μ_{Ai} and μ_{J_ν} as reversible measures, and the reversible processes coincide with determinantal processes $(\Xi(t), \mathbb{P}_{\sin})$ with the extended sine kernel (2.9), $(\Xi(t), \mathbb{P}_{\mathrm{Ai}})$ with the extended Airy kernel (2.10), and $(\Xi(t), \mathbb{P}_{\nu})$ with the extended Bessel kernel (2.11), respectively. The main purpose of this paper is to prove the