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Large deviation principle for some measure-valued processes

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Abstract

We establish a large deviation principle for the solutions of a class of stochastic partial differential equations with non-Lipschitz continuous coefficients. As an application, the large deviation principle is derived for super-Brownian motion and Fleming–Viot process.

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1. Introduction

Measure-valued processes (MVP) arise from many fields of applications including population growth models and genetics. We refer the reader to the books of Dawson [3], Etheridge [10], Perkins [18], and Li [15] for an introduction to this topic. Two of the most studied measure-valued processes are super-Brownian motion (SBM) and Fleming–Viot process (FVP). An interesting problem concerns the limiting behavior of these processes when the branching rate (for SBM) or the resampling rate (for FVP) ϵ, tends to zero. It is easy to see that the measure-valued processes,

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denoted by μ_t^ϵ, converge to a deterministic measure-valued process μ_t^0, and it is desirable to study the rate of convergence.

Another motivation for the study of the limit mentioned above is the scaling (on space/time/mass) of the MVPs. Namely, for an MVP X_t, we define the scaled process X_t^K by

$$X_t^K(B) \equiv K^{-1} X_{Kt}(KB), \quad \forall\, B \in \mathcal{B}(\mathbb{R}).$$

When X is an SBM, the limit of X^K as $K \to \infty$ was given by Dawson and Fleischmann [6] and the convergence rate was studied by Fleischmann and Kaj [13]. It is well-known that when X is an SBM with diffusion coefficient κ and branching rate 1, X^K is an SBM with diffusion coefficient κK^{-1} and branching rate K^{-1}. If we take $\kappa = K$, then the limit of X^K as $K \to \infty$ is equivalent to that of the SBM as the branching rate tends to 0.

The large deviation principle (LDP) is a very useful tool for the study of convergence rate. Roughly speaking, the goal of the LDP is to determine the rate $R(\delta) > 0$, for any $\delta > 0$ such that as $\epsilon \to 0$,

$$P\left(\rho\left(\mu^\epsilon, \mu^0\right) > \delta\right) \approx \exp\left(-\epsilon^{-1} R(\delta)\right), \tag{1}$$

for a suitable distance, ρ in $C([0, 1]; \mathcal{M}_F(\mathbb{R}))$, the state space of the MVP, where $\mathcal{M}_F(\mathbb{R})$ is the set of finite measures μ on $\mathbb{R}$. We refer the reader to the books of Dembo and Zeitouni [7], Deuschel and Stroock [8], and Dupuis and Ellis [9] for more background on this subject. Due to a technical reason which will become clear later, we have to derive our LDP in a double quotient space of $C([0, 1]; \mathcal{M}_F(\mathbb{R}))$ whose topology is coarser than the original one.

LDP for MVP has been studied by many authors. Fleischmann and Kaj [13] proved the LDP for SBM for a fixed time t. Later on, sample path LDP for SBM was derived independently by Fleischmann et al. [12], and Schied [19] while the rate function was expressed by a variational form. To obtain an explicit expression for the rate function, [12] assumed a *local blow-up condition* which was not proven. On the other hand, [19] obtained the explicit expression of the rate function when the term representing the movements of the particles also tends to zero. The local blow-up condition of [12] was recently removed by Xiang for SBM with finite and infinite initial measure, [25,24] respectively, and the same explicit expression was derived. Fleischmann and Xiong [14] proved an LDP for catalytic SBM with a single point catalyst. The successes of the LDP for SBM depend on the branching property of this process. This property implies the weak LDP directly, and hence the problem diminishes to showing the exponential tightness of SBM, which yields the LDP, and identifying its rate function. Making use of the Brownian snake representation introduced by LeGall, Serlet [20,21] also obtained large deviation estimates for SBM.

Since FVP does not possess the branching property, the derivation of LDP depends on new ideas. Dawson and Feng [4,5], and Feng and Xiong [11] considered the LDP for FVP when the mutation is neutral. In [5], LDP was shown to hold when the process remains in the interior of the simplex, and in [3] the authors proved that if the process starts from the interior, it will not reach the boundary. On the other hand, authors in [11] focused on the singular case when the process starts from the boundary. For non-neutral case, Xiang and Zhang [26] derived an LDP for FVP when the mutation operator also tends to zero by projecting to the finite dimensional case.

The goal of this paper is to study LDP for MVP, with SBM and FVP as special cases. Comparing our LDP for SBM with that obtained in [12,19,24], our paper offers a different approach. Our LDP for FVP contributes to the literature, by not requiring the neutrality and vanishing of mutation. We note that our method only applies to the case of superprocesses with spatial dimension one. The key in our proof is the representation of the MVPs as the unique strong solution