Available online at www.sciencedirect.com

ScienceDirect

Stochastic Processes and their Applications 125 (2015) 1127–1147

stochastic processes and their applications

www.elsevier.com/locate/spa

Linear Multifractional Stable Motion: Representation via Haar basis

Julien Hamonier

Laboratoire de Physique, CNRS UMR 5672, ENS Lyon 46, allée d'Italie, F-69364 Lyon cedex, France

Received 11 September 2013; received in revised form 10 October 2014; accepted 10 October 2014
Available online 22 October 2014

Abstract

The goal of this paper is to provide a wavelet series representation for Linear Multifractional Stable Motion (LMSM). Instead of using Daubechies wavelets, which are not given in closed form, we use a Haar wavelet, thus yielding a more explicit expression than that in Ayache and Hamonier (in press).

The main ingredient of this construction is a Haar expansion of the integrands which define the high and low frequency components of the $\mathcal{S}\alpha\mathcal{S}$ random field X generating LMSM. Then, by using Abel summation, we show that these series are a.s. convergent in the space of continuous functions. Finally, we determine their a.s. rates of convergence in the latter space.

In the end, the representations of the high and low frequency components of X, provide a new method for simulating the high and low frequency components of LMSM. Moreover, this new way is faster than the way based on Fast Fourier Transform (Le Guevel and Levy-vehel, 2012; Stoev and Taqqu, 2004).

MSC: 60G22; 60G52; 41A30; 41A58

Keywords: Approximation of processes; Linear Fractional and Multifractional Stable Motions; Wavelet series representations; Haar system

1. Introduction

Over the last decade, there has been a growing interest in probabilistic models based on fractional and multifractional processes. These parametric families of stochastic processes convey a

E-mail address: julien.hamonier@gmail.com.

http://dx.doi.org/10.1016/j.spa.2014.10.013
0304-4149/

convenient framework for modelling in several fields such as Internet traffic and Finance. The models are natural extensions of the well-known Gaussian fractional Brownian motion (fBm), which has stationary increments and is self-similar with self-similar exponent $H \in (0, 1)$. One of the best-known extensions of fBm to the setting of heavy-tailed stable distributions is the so-named Linear Fractional Stable Motion (LFSM) (see [13,6]). The latter also has stationary increments and is self-similar with exponent $H \in (0, 1)$. Nevertheless, its law depends on a second parameter, denoted by $\alpha \in (0, 2)$, which controls the tail heaviness of the finite-dimensional distributions of LFSM.

In order to overcome some limitations due to the stationarity of increments and the constancy of the self-similarity exponent, Linear Multifractional Stable Motions (LMSMs) were introduced in [16,15]. LMSMs are good candidate models for describing some features of Internet traffic such as, burstiness, which is the presence of rare but extremely busy periods of activity.

To define LMSM, we need to establish the following assumptions that will be used throughout the article:

(A1) $\mathrm{Z}_\alpha(ds)$ is an independently scattered symmetric α-stable ($\mathcal{S}\alpha\mathcal{S}$) random measure on $\mathbb{R}$, with Lebesgue control measure [13].
(A2) We assume that $\alpha \in (1, 2)$. Stoev and Taqqu [15] showed that this is a necessary condition for the a.s. continuity of the paths of LMSM.
(A3) $H(\cdot)$ denotes an arbitrary deterministic continuous function defined on the real line and with values in an arbitrary fixed compact interval $[\underline{H}, \overline{H}] \subset (1/\alpha, 1)$;

A LMSM, denoted by $Y = \{Y(t) : t \in \mathbb{R}\}$, with functional Hurst parameter $H(\cdot)$, as

$$Y(t) := X(t, H(t)), \quad t \in \mathbb{R}. \tag{1.1}$$

In (1.1), $X = \{X(u, v) : (u, v) \in \mathbb{R} \times (1/\alpha, 1)\}$ is a $\mathcal{S}\alpha\mathcal{S}$ random field such that for every $(u, v) \in \mathbb{R} \times (1/\alpha, 1)$,

$$X(u, v) := \int_{\mathbb{R}} \left\{ (u - s)_+^{v-1/\alpha} - (-s)_+^{v-1/\alpha} \right\} \mathrm{Z}_\alpha(ds), \tag{1.2}$$

where for all $(s, \kappa) \in \mathbb{R}^2$,

$$(s)_+^\kappa = s^\kappa \quad \text{if } s > 0 \quad \text{and} \quad (s)_+^\kappa = 0 \text{ else.} \tag{1.3}$$

A modification of the high frequency component of X is the $\mathcal{S}\alpha\mathcal{S}$ random field $X_1 = \{X_1(u, v) : (u, v) \in \mathbb{R} \times (1/\alpha, 1)\}$, defined for each $(u, v) \in \mathbb{R} \times (1/\alpha, 1)$ as

$$X_1(u, v) := \int_0^{+\infty} (u - s)_+^{v-1/\alpha} \mathrm{Z}_\alpha(ds). \tag{1.4}$$

A modification of the low frequency component of X is the $\mathcal{S}\alpha\mathcal{S}$ stochastic field $X_2 = \{X_2(u, v) : (u, v) \in \mathbb{R} \times (1/\alpha, 1)\}$, defined for each $(u, v) \in \mathbb{R} \times (1/\alpha, 1)$, as

$$X_2(u, v) := \int_{-\infty}^0 \left\{ (u - s)_+^{v-1/\alpha} - (-s)_+^{v-1/\alpha} \right\} \mathrm{Z}_\alpha(ds). \tag{1.5}$$

It is worth noticing that these fields display rather different properties. Moreover, in view of (1.2), (1.4) and (1.5), one has for all $(u, v) \in \mathbb{R} \times (1/\alpha, 1)$

$$X(u, v) = X_1(u, v) + X_2(u, v) \quad \text{a.s.}$$