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Extreme slowdowns for one-dimensional excited random walks

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Abstract

We study the asymptotics of the probabilities of extreme slowdown events for transient one-dimensional excited random walks. That is, if $\{X_n\}_{n\geq 0}$ is a transient one-dimensional excited random walk and $T_n = \min\{k : X_k = n\}$, we study the asymptotics of probabilities of the form $P(X_n \leq n^\gamma)$ and $P(T_{n^\gamma} \geq n)$ with $\gamma < 1$. We show that there is an interesting change in the rate of decay of these extreme slowdown probabilities when $\gamma < 1/2$.

MSC: primary 60K35; secondary 60F10; 60J15; 60K37

Keywords: Excited random walk; Large deviations

1. Introduction and main results

Excited random walks are a model of self-interacting random walks where the transition probabilities are a function of the local time of the random walk at the current location. The model of excited random walks was first introduced by Benjamini and Wilson in [3], but has since been generalized by Zerner [17] and more recently by Kosygina and Zerner [12]. For

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the case of one-dimensional excited random walks, the model can be described as follows. A *cookie environment* is an element $\omega = \{\omega(x, j)\}_{x\in\mathbb{Z},\, j\geq 1} \in [0,1]^{\mathbb{Z}\times\mathbb{N}} =: \Omega$. For a fixed cookie environment ω we can define a self-interacting random walk on $\mathbb{Z}$ so that on the jth visit of the random walk to the site x, the random walk steps to the right with probability $\omega(x, j)$ and to the left with probability $1 - \omega(x, j)$. That is, $\{X_n\}_{n\geq 0}$ is a stochastic process with law P_ω such that

$$\begin{aligned} P_\omega(X_{n+1} = X_n + 1 \mid \mathcal{F}_n) &= 1 - P_\omega(X_{n+1} = X_n - 1 \mid \mathcal{F}_n) \\ &= \omega(X_n, \#\{k \leq n : X_k = X_n\}), \end{aligned}$$

where $\mathcal{F}_n = \sigma(X_0, X_1, \ldots, X_n)$. One can start the excited random walk at any $x \in \mathbb{Z}$, but in this paper we will always start the excited random walks at $X_0 = 0$.

We will allow the cookie environments to be random, chosen from a distribution $\mathbb{P}$ on the space Ω of cookie environments (equipped with the standard product topology). The distribution P_ω of the random walk in a fixed cookie environment is the *quenched* law of the random walk. Since the environment ω is random, P_ω is a conditional probability distribution, and the *averaged* law P of the excited random walk is defined by averaging the quenched law over all environments. That is, $P(\cdot) = \mathbb{E}[P_\omega(\cdot)]$, where $\mathbb{E}$ denotes expectation with respect to the distribution $\mathbb{P}$ on environments.

The terminology "cookie environment" is traced back to Zerner's paper [17] where he envisioned a stack of "cookies" at each site. Upon each visit to a site, the random walker eats a cookie (removing it from the stack) and the cookie induces a specific drift on the random walker.[1] Most of the results for one-dimensional excited random walks are under the assumption of a bounded number of cookies per site and i.i.d. stacks of cookies. More specifically, we will assume the following.

Assumption 1. There exists an $M < \infty$ such that $\mathbb{P}(\omega \in \Omega_M) = 1$, where

$$\Omega_M = \{\omega \in \Omega : \omega(x, j) = 1/2 \text{ for all } x \in \mathbb{Z},\ j > M\}.$$

Assumption 2. The distribution $\mathbb{P}$ on cookie environments ω is such that $\{\omega(x, \cdot)\}_{x\in\mathbb{Z}}$ is i.i.d. under the measure $\mathbb{P}$.

Assumption 1 is said to be the assumption of M cookies per site because one imagines that after the M cookies at a site have been removed, upon further returns to that site there are no cookies to "excite" the walk and so the walk moves as a simple symmetric random walk. Note that Ω_M is obviously isomorphic to the space $[0,1]^{\mathbb{Z}\times M}$.

In addition to the above assumptions on the cookie environments, we will also need the following non-degeneracy assumption on the cookie environments.

Assumption 3. The distribution $\mathbb{P}$ on cookie environments is such that

$$\mathbb{E}\left[\prod_{j=1}^{M} \omega(0, j)\right] > 0, \quad \text{and} \quad \mathbb{E}\left[\prod_{j=1}^{M} (1 - \omega(0, j))\right] > 0.$$

[1] For this reason excited random walks are also sometimes called "cookie random walks".