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Multilevel Monte Carlo simulation for Lévy processes based on the Wiener–Hopf factorisation

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Abstract

In Kuznetsov et al. (2011) a new Monte Carlo simulation technique was introduced for a large family of Lévy processes that is based on the Wiener–Hopf decomposition. We pursue this idea further by combining their technique with the recently introduced multilevel Monte Carlo methodology. Moreover, we provide here for the first time a theoretical analysis of the new Monte Carlo simulation technique in Kuznetsov et al. (2011) and of its multilevel variant for computing expectations of functions depending on the historical trajectory of a Lévy process. We derive rates of convergence for both methods and show that they are uniform with respect to the "jump activity" (e.g. characterised by the Blumenthal–Getoor index). We also present a modified version of the algorithm in Kuznetsov et al. (2011) which combined with the multilevel methodology obtains the optimal rate of convergence for general Lévy processes and Lipschitz functionals. This final result is only a theoretical one at present, since it requires independent sampling from a triple of distributions which is currently only possible for a limited number of processes.

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Keywords: Wiener–Hopf decomposition; Monte Carlo simulation; Multilevel Monte Carlo; Lévy processes; Barrier options

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1. Introduction

A classical problem in mathematical finance deals with the ability to compute $\mathbb{E}[F(X)]$, where $X := \{X_s : s \in [0,t]\}$ is a stochastic process which models the underlying risky asset and F is a payoff function which may depend on the historical path of X. This is the typical setting for pricing a variety of exotic options in finance such as look back and barrier options. Starting with the early work of Madan and Seneta [30], a class of processes playing the role of X that have found prominence in this respect is the class of Lévy processes. An extensive overview of their position in mathematical finance can be found in the books [6,11,33,34]. More recently Lévy processes have also been extensively used in modern insurance risk theory; see for example Asmussen and Albrecher [2] and Klüppelberg et al. [25]. In insurance mathematics, it is the Lévy process itself which models the surplus wealth of an insurance company until its ruin. There are also extensive applications of Lévy processes in queuing theory, genetics and mathematical biology as well as in stochastic differential equations (see e.g. [8,14,16,23]).

In both financial and insurance settings, a key quantity of generic interest is the joint law of the current position and the running maximum of a Lévy process at a fixed time, if not the individual marginals associated with the latter bivariate law. Consider the following example. If we define $\overline{X}_t = \sup_{s \le t} X_s$ then the pricing of barrier options boils down to evaluating expectations of the form $\mathbb{E}[f(x + X_t)\mathbf{1}_{\{x+\overline{X}_t > b\}}]$ for some threshold $b > 0$. Indeed if $f(x) = (K - \mathrm{e}^x)^+$ and $K > 0$, then the latter expectation is related to the value of an "up-and-in" put. In credit risk one is predominantly interested in the quantity $\widehat{\mathbb{P}}(\overline{X}_t < x)$ as a function in x and t, where $\widehat{\mathbb{P}}$ is the law of the dual process $-X$. Indeed it is a functional of the latter probabilities that gives the price of a credit default swap not to mention the recently introduced financial instruments known as convertible contingencies (CoCos). See for example the recent book of Schoutens and Cariboni [34] as well as Corcuera et al. [12]. One is similarly interested in $\widehat{\mathbb{P}}(\overline{X}_t \ge x)$ in ruin theory, since these probabilities are also equivalent to the finite-time ruin probabilities; cf. Asmussen and Albrecher [2].

A widely used approach to compute expectations of functions depending on the historical trajectory of a Lévy process over the time horizon, say $[0,t]$, is to approximate the path by a random walk with n steps, each step covering t/n units of time, and therewith to perform a Monte Carlo (MC) simulation. Giles [18,19] introduced an adaptation of the straightforward MC methodology, the multilevel Monte Carlo method (MLMC), which is especially suited to the scenario we are interested in, but in the case that X is a pure diffusion process. Very recently, there has been increasing attention to the MLMC method also in the setting of Lévy processes, see Giles and Xia [20] for the jump–diffusion setting, and Dereich [14] and Dereich and Heidenreich [15] for more general Lévy processes. Generally speaking all these methods share a common approach, which consists in constructing an embedded sequence of grids that are made up of a mixture of deterministic and random points. The random points in these grids deal with the large jumps of the Lévy process and the deterministic points deal with the "small movements", that is to say, the diffusive part and/or the small jumps.

In this paper we consider an alternative method based entirely on a random grid. In particular we shall introduce an adaptation of the MLMC method based on a very recently introduced technique for performing MC simulations that appeals to the so-called Wiener–Hopf factorisation for one-dimensional Lévy processes. This last technique is called the Wiener–Hopf Monte Carlo (WHMC) simulation method and it was introduced in Kuznetsov et al. [28]. We will denote the