

Joint temporal and contemporaneous aggregation of random-coefficient AR(1) processes

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Abstract

We discuss joint temporal and contemporaneous aggregation of N independent copies of AR(1) process with random-coefficient $a \in [0, 1)$ when N and time scale n increase at different rate. Assuming that a has a density, regularly varying at $a = 1$ with exponent $-1 < \beta < 1$, different joint limits of normalized aggregated partial sums are shown to exist when $N^{1/(1+\beta)}/n$ tends to (i) ∞ , (ii) 0, (iii) $0 < \mu < \infty$. The limit process arising under (iii) admits a Poisson integral representation on $(0, \infty) \times C(\mathbb{R})$ and enjoys ‘intermediate’ properties between fractional Brownian motion limit in (i) and sub-Gaussian limit in (ii).
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1. Introduction

Since macroeconomic time series are obtained by aggregation of microeconomic variables, an important issue in econometrics is establishing the relationship between individual (micro) and aggregate (macro) models. One of the simplest aggregation schemes deals with contemporaneous aggregation of N independent copies $\{X_i(t), t \geq 1\}$, $i = 1, \dots, N$ of stationary random-coefficient AR(1) process

$$X(t) = aX(t-1) + \varepsilon(t), \quad t \geq 1, \quad (1.1)$$

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with standardized i.i.d. innovations $\{\varepsilon(t)\}$ and a random coefficient $a \in [0, 1)$, independent of $\{\varepsilon(t)\}$ and such that $E(1 - a)^{-1} < \infty$. The limit aggregated process

$$N^{-1/2} \sum_{i=1}^N X_i(t) \rightarrow_{\text{fdd}} \mathcal{X}(t), \quad t \geq 1, \quad (1.2)$$

exists, in the sense of weak convergence of finite-dimensional distributions, and is a Gaussian process with zero mean and covariance function

$$E[\mathcal{X}(0)\mathcal{X}(t)] = E[X(0)X(t)] = E\left[\frac{a^t}{1-a^2}\right], \quad t \geq 0. \quad (1.3)$$

Granger [9] observed that for a particular type of beta-distributed random coefficient a , the processes $\{X(t)\}$ and $\{\mathcal{X}(t)\}$ may have slowly decaying autocovariance functions similarly as in the case of ARFIMA models while normalized partial sums of $\{\mathcal{X}(t)\}$ tend to a fractional Brownian motion. Further results on aggregation of autoregressive models with finite variance were obtained in Gonçalves and Gouriéroux [8], Zaffaroni [30], Oppenheim and Viano [19], Celov et al. [3] and other papers. In economic interpretation, individual processes $\{X_i(t)\}$, $i = 1, \dots, N$ in (1.2) are obtained by random sampling from a huge and heterogeneous ‘population’ of independent ‘microagents’, each evolving according to a short memory AR(1) process with its own deterministic parameter $a \in [0, 1)$, the population being characterized by the distribution (frequency) of a across the population. Thus, aggregation of (randomly sampled) short memory processes may provide an explanation of long memory in observed macroeconomic time series. See also [2, p. 85], [30], [31, p. 238].

The present paper discusses the limit behavior of sums

$$S_{N,n}(\tau) := \sum_{i=1}^N \sum_{t=1}^{\lfloor n\tau \rfloor} X_i(t), \quad \tau \geq 0, \quad (1.4)$$

where $\{X_i(t)\}$, $i = 1, \dots, N$ are the same as in (1.2). The sum in (1.4) represents joint temporal and contemporaneous aggregate of N individual AR(1) evolutions (1.1) at time scale n . The main object of our paper is the joint aggregation limit $\lim_{N,n \rightarrow \infty} A_{N,n}^{-1} S_{N,n}(\tau)$, in distribution, where $A_{N,n}$ are some normalizing constants and both N and n increase to infinity, possibly at different rate. We also discuss the iterated limits of $A_{N,n}^{-1} S_{N,n}(\tau)$ when first $n \rightarrow \infty$ and then $N \rightarrow \infty$, and vice-versa. Related problems for some network traffic models were studied in Willinger et al. [29], Taqqu et al. [28], Mikosch et al. [18], Gaigalas and Kaj [7], Pipiras et al. [20], Dombry and Kaj [4] and other papers. In these papers, the role of AR(1) processes $\{X_i(t)\}$ in (1.4) is played by independent and centered ON/OFF processes, integrated renewal or renewal-reward processes, or $M/G/\infty$ queues with heavy-tailed activity periods.

Let us describe the main results of this paper. Similarly as in [30,22], we assume that the r.v. $a \in [0, 1)$ in (1.1), or the mixing distribution, has a probability density $\phi(x)$ such that

$$\phi(x) = \psi(x)(1-x)^\beta, \quad x \in [0, 1), \quad (1.5)$$

where $\beta > -1$ and ψ is an integrable function on $[0, 1)$ having a limit $\lim_{x \rightarrow 1} \psi(x) = \psi_1 > 0$. Under the above condition with $0 < \beta < 1$, it easily follows from the Tauberian theorem [5, Chapter 13, Section 5, Theorem 3] that the covariance in (1.3) decays as $ct^{-\beta}$, $t \rightarrow \infty$, with $c = (\psi_1/2)\Gamma(\beta)$, implying that partial sums of the Gaussian process $\{\mathcal{X}(t)\}$ in (1.2) normalized by n^H , $H := 1 - \frac{\beta}{2} \in (\frac{1}{2}, 1)$ tend to a fractional Brownian motion B_H with Hurst parameter

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