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Uniform concentration inequality for ergodic diffusion processes observed at discrete times[☆]

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Abstract

In this paper a concentration inequality is proved for the deviation in the ergodic theorem for diffusion processes in the case of discrete time observations. The proof is based on geometric ergodicity of diffusion processes. We consider as an application the nonparametric pointwise estimation problem of the drift coefficient when the process is observed at discrete times.
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1. Introduction

We consider the process $(y_t)_{t\geq 0}$ governed by the stochastic differential equation

$$\mathrm{d}y_t = S(y_t)\,\mathrm{d}t + \sigma(y_t)\,\mathrm{d}W_t, \tag{1.1}$$

where $(W_t, \mathcal{F}_t)_{t\geq 0}$ is a standard Wiener process and y_0 is an initial condition.

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Before we state the main result of the paper on the concentration inequality we start with a nonparametric estimation problem for the process (1.1), where this kind of inequality appears. Suppose that the coefficients S, σ are unknown and the process (y_t) is observed on the interval $[0, T]$ at discrete times. We consider the pointwise estimation problem for the function S at a fixed point $x_0 \in \mathbb{R}$ (i.e. $S(x_0)$) given the observations of the process (1.1)

$$(y_{t_j})_{1\le j\le N}, \quad 0 \le t_j \le T, \tag{1.2}$$

where $t_j = j\delta$, $N = [T/\delta]$ and δ is some positive fixed observation frequency which will be specified later. Usually, for this problem one uses kernel estimators $\widehat{S}_N(x_0)$ defined as

$$\widehat{S}_N(x_0) = \frac{\sum_{k=1}^{N} \psi_{h,x_0}(y_{t_k})\, \Delta y_{t_k}}{\sum_{k=1}^{N} \psi_{h,x_0}(y_{t_k})\, \Delta t_k}, \qquad \psi_{h,x_0}(y) = \frac{1}{h}\, \Psi\left(\frac{y-x_0}{h}\right), \tag{1.3}$$

where $\Psi(y)$ is a kernel function which is equal to zero for $|y| \ge 2$ and will be specified later, $0 < h < 1$ is a bandwidth, $\Delta y_{t_k} = y_{t_k} - y_{t_{k-1}}$ and $\Delta t_k = \delta$.

The main difficulty in studying this estimator is that the denominator is a random variable. In particular, to obtain the convergence rate of this estimator one has to study the asymptotic behavior of the denominator; more precisely, one needs to show that

$$\sum_{k=1}^{N} \psi_{h,x_0}(y_{t_k}) \Delta t_k \approx \pi_\vartheta(\psi_{h,x_0}) h T \quad \text{as } T \to \infty,$$

where

$$\pi_\vartheta(\psi_{h,x_0}) = \int_{\mathbb{R}} \psi_{h,x_0}(y)\, q_\vartheta(y)\, \mathrm{d}y \tag{1.4}$$

and q_ϑ is the invariant density defined in (2.2).

Unfortunately, the ergodic theorem does not permit us to obtain this kind of result because the times t_k and the bandwidth h depend on T. Usually, one obtains the desired property through concentration inequalities for the deviation in the ergodic theorem. The deviation is as follows:

$$D_T(\phi) = \sum_{k=1}^{N} \left(\phi(y_{t_k}) - \pi_\vartheta(\phi)\right) \Delta t_k, \tag{1.5}$$

where ϕ is some function which may depend on T, for example, $\phi(\cdot) = \psi_{h,x_0}(\cdot)$. The concentration inequality provides the limit behavior of tail probabilities; more precisely, it shows that, for any $\varepsilon > 0$ and for any $m > 0$, uniformly over ϑ,

$$\lim_{T\to\infty} T^m\, \mathbf{P}_\vartheta\left(|D_T(\psi_{h,x_0})| > \varepsilon T\right) = 0, \tag{1.6}$$

where $\mathbf{P}_\vartheta$ is the law of the process $(y_t)_{t\ge 0}$ under the coefficients $\vartheta = (S, \sigma)$. Usually, to get properties of type (1.6) one needs to establish an exponential inequality for the deviation (1.5).

There are a number of papers devoted to concentration inequalities for functions of independent random variables (we refer the reader to [2] and references therein), and for functions of dependent random variables (see [4,5,16]). For Markov chains such inequalities were obtained in [1]. For continuous time Markov processes an exponential concentration

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