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# Multi-operator scaling random fields☆

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**Abstract**

In this paper, we define and study a new class of random fields called harmonizable multi-operator scaling stable random fields. These fields satisfy a local asymptotic operator scaling property which generalizes both the local asymptotic self-similarity property and the operator scaling property. Actually, they locally look like operator scaling random fields, whose order is allowed to vary along the sample paths. We also give an upper bound of their modulus of continuity. Their pointwise Hölder exponents may also vary with the position $x$ and their anisotropic behavior is driven by a matrix which may also depend on $x$.


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*Keywords:* Gaussian and stable random fields; Local operator scaling property; Hölder regularity

## 1. Introduction

Self-similar random processes and fields are required to model numerous natural phenomena, e.g., in internet traffic, hydrology, geophysics or financial markets; see for instance [27,18,1]. A very important class of such fields is given by fractional stable random fields (see [25]). In particular, the well-known fractional Brownian field $B_H$ is a Gaussian $H$-self-similar random field with

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stationary increments. It is an isotropic generalization of the famous fractional Brownian motion [19,12]. Self-similar isotropic $\alpha$-stable fields have been extensively used to propose an alternative to Gaussian modeling (see [21,27] for instance) to mimic heavy-tailed persistent phenomena.

However, isotropy property is a serious drawback for many applications in medicine [8], geophysics [22,9] and hydrology [5], just to mention a few. Recently, an important class of anisotropic random fields has been studied in [7]. These fields are anisotropic generalizations of self-similar stable random fields. They satisfy an operator scaling property which generalizes the classical self-similarity property. More precisely, for $E$, a real $d \times d$ matrix whose eigenvalues have positive real parts, a scalar valued random field $(X(x))_{x\in\mathbb{R}^d}$ is called *operator scaling* of order $E$ and $H > 0$ if, for every $c > 0$,

$$\{X(c^E x); x \in \mathbb{R}^d\} \overset{(fdd)}{=} \{c^H X(x); x \in \mathbb{R}^d\}, \tag{1}$$

where $\overset{(fdd)}{=}$ means equality of finite dimensional distributions and as usual $c^E = \exp(E \log c)$. Let us recall that the self-similarity property corresponds to the case where $E$ is the identity matrix. Let us also remark that up to considering the matrix $E/H$, we may and will assume, without loss of generality, that $H = 1$. The anisotropic behavior of operator scaling random fields with stationary increments is then driven by a matrix. In particular, when $\theta_j$ is an eigenvector of $E$ associated with the eigenvalue $\lambda_j$, any operator scaling random field for $E$ is $1/\lambda_j$-self-similar in direction $\theta_j$. Furthermore, the critical global and directional Hölder exponents of harmonizable operator scaling stable random fields are given by the eigenvalues of $E$ (see [7,6]). Let us emphasize that these exponents and the directions of self-similarity do not vary according to the position.

Moreover, the self-similarity is a global property which can be too restrictive for applications. Actually, numerous phenomena exhibit scale invariance that may vary according to the scale or to the position and are usually called multifractal (see [10,24,22] for examples). To allow more flexibility, [4] has introduced the local asymptotic self-similarity property. This property characterizes random fields that locally seem self-similar but whose local regularity properties evolve. Since then, many examples of locally asymptotically self-similar random fields have been introduced and studied, e.g., in [4,23,3,2,15,26].

In this paper, we introduce the local asymptotic operator scaling property which generalizes both the local asymptotic self-similarity property and the operator scaling property. A scalar valued random field $X$ is *locally asymptotically operator scaling at point $x$ of order $A(x)$* if

$$\lim_{\varepsilon\to 0^+} \left( \frac{X(x+\varepsilon^{A(x)}u) - X(x)}{\varepsilon} \right)_{u\in\mathbb{R}^d} \overset{(fdd)}{=} (Z_x(u))_{u\in\mathbb{R}^d}\,, \tag{2}$$

with $Z_x$ a non degenerate random field. Let us first remark that the local asymptotic self-similarity property of exponent $h(x)$ corresponds to the local asymptotic operator self-similarity of order $A(x) = I_d/h(x)$ with $I_d$ the identity matrix of order $d$. Moreover, operator scaling random fields of order $E$ are locally asymptotically operator scaling at point 0 of order $E$. Of course, if they also have stationary increments, they are locally asymptotically operator scaling at any point $x$. In addition, if (2) is fulfilled, the random field $Z_x$ is operator scaling of order $A(x)$. In other words, a local asymptotic multi-operator random field locally looks like an operator scaling random field, whose order is allowed to vary along the sample paths.

Then, we focus on harmonizable multi-operator scaling stable random fields, which generalize harmonizable operator scaling stable random fields. A harmonizable multi-operator scaling