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Berry–Esseen and Edgeworth approximations for the normalized tail of an infinite sum of independent weighted gamma random variables

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Abstract

Consider the sum $Z = \sum_{n=1}^{\infty} \lambda_n(\eta_n - \mathbb{E}\eta_n)$, where η_n are independent gamma random variables with shape parameters $r_n > 0$, and the λ_n's are predetermined weights. We study the asymptotic behavior of the tail $\sum_{n=M}^{\infty} \lambda_n(\eta_n - \mathbb{E}\eta_n)$, which is asymptotically normal under certain conditions. We derive a Berry–Esseen bound and Edgeworth expansions for its distribution function. We illustrate the effectiveness of these expansions on an infinite sum of weighted chi-squared distributions.

The results we obtain are directly applicable to the study of double Wiener–Itô integrals and to the "Rosenblatt distribution".

MSC: 60E05; 60E10; 60E99

Keywords: Berry–Esseen; Edgeworth expansions; Infinitely divisible distributions; Rosenblatt distribution

1. Introduction

We focus on the distribution of normalized tails of infinite weighted sums of *Gamma* random variables. Our results involve the interplay between the asymptotic behavior of the weights λ_n in the sum and the scales r_n of Gamma variables. We show that one cannot always expect a central limit theorem to hold. When the central limit theorem holds, we develop Berry–Esseen bounds and also Edgeworth expansions.

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The results we obtain are directly applicable to the study of double Wiener–Itô integrals since these can be represented as weighted sums of independent chi-squared variables which are special cases of Gamma variables.

The Rosenblatt distribution is a non-Gaussian distribution which often appears as a limit of normalized partial sums of random variables with long memory. The results obtained here play a key role in a major study of the "Rosenblatt distribution" in [21]. This is because a random variable with the Rosenblatt distribution can be represented as a double Wiener–Itô integral, and as we noted, these multiple integrals can be viewed as weighted sums of independent chi-squared random variables.

Consider then the random variable $\sum_{n=1}^{\infty} \lambda_n(\eta_n - \mathbb{E}\eta_n)$, where $\eta_n \sim \text{Gamma}(r_n, \theta_n)$ are independent gamma random variables with mean $\mathbb{E}\eta_n = r_n\theta_n$ and variance $\text{Var}\,\eta_n = r_n\theta_n^2$, where $r_n > 0$ and $\theta_n > 0$ are the shape and scale parameters, respectively. We may suppose without loss of generality that $\theta_n = r_n^{-1}$ for all n, by incorporating the extra parameters into the constants λ_n.[1]

We thus consider

$$Z = \sum_{n=1}^{\infty} \lambda_n(\eta_n - 1), \tag{1}$$

where

$$\eta_n \sim \text{Gamma}(r_n, 1/r_n)$$

are independent gamma with pdf

$$f_{\eta_n}(x) = \frac{r_n^{r_n}}{\Gamma(r_n)} x^{r_n - 1} e^{-r_n x}, \quad x > 0. \tag{2}$$

We suppose that $\{\lambda_n\}$ and $\{r_n\}$ are sequences of positive numbers such that

$$\sum_{n=1}^{\infty} \frac{\lambda_n^2}{r_n} < \infty. \tag{3}$$

With this setup, Z has mean zero and variance

$$\text{Var}\, Z = \sum_{n=1}^{\infty} \lambda_n^2 \text{Var}\,(\eta_n - 1) = \sum_{n=1}^{\infty} \frac{\lambda_n^2}{r_n}.$$

Of particular interest is the case where $r_n = r$ is constant and $\lambda_n \sim n^{-\alpha/2}\ell(n)$, where $\alpha > 1$ and ℓ is slowly varying as $n \to \infty$. The restriction $\alpha > 1$ ensures $\sum \lambda_n^2/r_n = (1/r)\sum \lambda_n^2 < \infty$ but allows for cases when either $\sum \lambda_n = \infty$ or $\sum \lambda_n < \infty$.

[1] Since the characteristic function of $\eta \sim \text{Gamma}(r, \theta)$ is $\varphi_\eta(u) = (1 - iu\theta)^{-r}$ and that of $\hat{\eta} \sim \text{Gamma}(r, 1/r)$ is $\varphi_{\hat{\eta}}(u) = (1 - iu/r)^{-r}$, we have $\eta \overset{d}{=} \theta r \hat{\eta}$ and thus

$$\sum_{n=1}^{\infty} \lambda_n(\eta_n - \mathbb{E}\eta_n) = \sum_{n=1}^{\infty} \lambda_n(\eta_n - r_n\theta_n) \overset{d}{=} \sum_{n=1}^{\infty} \lambda_n r_n \theta_n(\hat{\eta}_n - 1) = \sum_{n=1}^{\infty} \hat{\lambda}_n(\hat{\eta}_n - 1)$$

by setting $\hat{\lambda}_n = \lambda_n r_n \theta_n$. In the text, we denote from now on $\hat{\lambda}_n$ and $\hat{\eta}_n$ by λ_n and η_n respectively.