

Impulse control problem on finite horizon with execution delay

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Abstract

We consider impulse control problems in finite horizon for diffusions with decision lag and execution delay. The new feature is that our general framework deals with the important case when several consecutive orders may be decided before the effective execution of the first one. This is motivated by financial applications in the trading of illiquid assets such as hedge funds. We show that the value functions for such control problems satisfy a suitable version of dynamic programming principle in finite dimension, which takes into account the past dependence of state process through the pending orders. The corresponding Bellman partial differential equations (PDE) system is derived, and exhibit some peculiarities on the coupled equations, domains and boundary conditions. We prove a unique characterization of the value functions to this nonstandard PDE system by means of viscosity solutions. We then provide an algorithm to find the value functions and the optimal control. This easily implementable algorithm involves backward and forward iterations on the domains and the value functions, which appear in turn as original arguments in the proofs for the boundary conditions and uniqueness results.

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1. Introduction

In this paper, we consider a general impulse control problem in finite horizon of a diffusion process X , with intervention lag and execution delay. This means that we may intervene on the diffusion system at any times τ_i separated at least by some fixed positive lag h , by giving some impulse ξ_i based on the information at τ_i . However, the execution of the impulse decided at τ_i is carried out with delay mh , $m \geq 1$, i.e. it is implemented at time $\tau_i + mh$, moving the system from $X_{(\tau_i+mh)^-}$ to $\Gamma(X_{(\tau_i+mh)^-}, \xi_i)$. The objective is to maximize over impulse controls $(\tau_i, \xi_i)_i$ the expected total profit on finite horizon T , of the form

$$\mathbb{E} \left[\int_0^T f(X_t) dt + g(X_T) + \sum_{\tau_i + mh \leq T} c(X_{(\tau_i+mh)^-}, \xi_i) \right].$$

Such formulations appear naturally in decision-making problems in economics and finance. In many situations, firms or investors face regulatory delays (delivery lag), which may be significant, and thus need to be taken into account when management strategies are decided in an uncertain environment. Problems where firm's investment are subject to delivery lag can be found in the real options literature, for example in [2,1]. In financial market context, execution delay is related to liquidity risk (see e.g. [15]), and occurs with transaction, which requires heavy preparatory work as for hedge funds. Indeed, hedge funds frequently hold illiquid assets, and need some time to find a counterpart to buy or sell them. Furthermore, this notice period gives the hedge fund manager a reasonable investment horizon.

From a mathematical viewpoint, it is well known that impulse control problems without delay, i.e. $m = 0$, lead to variational partial differential equations (PDE), see e.g. the books [5,11]. Impulse control problems in the presence of delay were studied in [14] for $m = 1$, that is when no more than one pending order is allowed at any time. In this case, it is shown that the delay problem may be transformed into a no-delay impulse control problem. The paper [4] also considers the case $m = 1$, but when the value of the impulse is chosen at the time of execution, and on infinite horizon, and these two conditions are crucial in the proposed probabilistic resolution. We mention also the works [3] and recently [12], which study impulse problems in infinite horizon with arbitrary number of pending orders, but under restrictive assumptions on the controlled state process, like (geometric) Lévy process for X and (multiplicative) additive intervention operator Γ . In this case, the problem is reduced to a finite-dimensional one where the value functions with pending orders are directly related to the value function without order.

The main contribution of this paper is to provide a theory of impulse control problems with delay on finite horizon in a fairly general diffusion framework that deals with the important case in applications when the number of pending orders is finite, but not restricted to one, i.e. $m \geq 1$. Our chief goal is to obtain a unique tractable PDE characterization of the value functions for such problems. As usual in stochastic control problems, the first step is the derivation of a dynamic programming principle (DPP). We show a suitable version of DPP, which takes into account the past dependence of the controlled diffusion via the finite number of pending orders. The corresponding Bellman PDE system reveals some nonstandard features both on the form of the differential operators and their domains, and on the boundary conditions. Following the standard approach to stochastic control, we prove that the value functions are viscosity solutions to this Bellman PDE system, and we also state comparison principles, which allows us to obtain a unique PDE characterization. From this PDE representation, we provide an easily implemented algorithm to compute the value functions, and the optimal impulse control as byproducts. In this

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