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Power utility maximization under partial information: Some convergence results[☆]

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Abstract

In this paper we consider the power utility maximization problem under partial information in a continuous semimartingale setting. Investors construct their strategies using the available information, which possibly may not even include the observation of the asset prices. Resorting to stochastic filtering, the problem is transformed into an equivalent one, which is formulated in terms of observable processes. The value process, related to the equivalent optimization problem, is then characterized as the unique bounded solution of a semimartingale backward stochastic differential equation (BSDE). This yields a unified characterization for the value process related to the power and exponential utility maximization problems, the latter arising as a particular case. The convergence of the corresponding optimal strategies is obtained by means of BSDEs. Finally, we study some particular cases where the value process admits an explicit expression.

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Keywords: Backward stochastic differential equation; Semimartingale market model; Power utility maximization problem; Partial information

1. Introduction

We consider an incomplete financial model, consisting of a bond and a risky asset with a continuous semimartingale (returns) dynamics. In this setup we address the problem of

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maximization of the expected utility from terminal wealth, assuming that the flow of observable events does not contain all market information. We refer to this situation as partial information, but in contrast to most papers dealing with restricted information, we will assume that the agents might not observe all information coming from the traded asset. This assumption is well justified from a practical point of view since the investors often need to take decisions with information gaps or delays. Besides, from a mathematical point of view, such a situation is more complicated to handle.

In this paper we deal with the issue of finding the best investment strategy, when trading takes place on a finite interval $[0, T]$ and, moreover, the quality is measured by the expected power utility of its terminal wealth. A filtration G smaller than the one of the full market $\mathcal{A}$ represents the information available to the investor.

Let R denote the process of returns associated to the risky asset, S the price process, i.e. $\mathrm{d}S_t = S_t \mathrm{d}R_t$, and let π represent the proportion of wealth invested in the asset. Suppose the non-risky investment pays no interest. $X_t^\pi = x + \int_0^t X_u^\pi \pi_u \mathrm{d}R_u$ describes the wealth process, which starts from an initial capital x and evolves according to the self-financing strategy π. Since X_t^π can be written in exponential form, in mathematical terms, the problem can be stated as follows

$$\text{maximize} \quad E\left[\frac{x^p \mathcal{E}_T^p(\pi \cdot R)}{p}\right] \quad \text{over all } \pi \in \Pi(G), \tag{1}$$

where $\mathcal{E}_t(X)$ denotes the Doléans–Dade exponential of X and $\Pi(G)$ a certain class of G adapted strategies to be subsequently defined.

Utility maximization problems under partial information have been considered under various setups in the literature. Mainly, (see, e.g., [12,9,21,25]) the problem was studied for market models where only stock prices are observed, while the drift cannot be directly observed, i.e. under the hypothesis $F^S \subseteq G$.

We include the case when the flow of observable events G does not necessarily contain all information on prices of the underlying asset, i.e., when S, as well as R, is not a G-semimartingale in general. Such an approach, in the context of exponential and mean variance hedging, was considered respectively in [14] and [16] (see also [23,4,20] for the mean variance hedging problem under partial information). We show that the initial problem (1) is equivalent to another maximization problem written in terms of the filtered terminal wealth. The main contribution of this paper is twofold: first it solves the problem of partial information for power utilities giving an explicit expression for optimal strategies; such a result is obtained by characterizing the value process related to the equivalent problem in terms of a backward stochastic differential equation (BSDE). This provides a unified characterization for both problems (power and exponential, see [14,6] for related results). Second, the convergence of the optimal strategies related to the power utility problem to the one of the exponential is established; this is achieved exploiting some kind of monotonicity properties of the solutions of the BSDEs.

In the case of full information, resorting to duality, the analogue convergence of strategies was derived in [1,11] considering continuous semimartingales and Lévy processes, respectively. The paper is organized as follows. Section 2 lays out the model and describes the main result of the paper. In Section 3 we find a characterization of the value process in terms of a BSDE with quadratic growth, whereas in Section 4 the convergence of the optimal strategies is studied. In Section 5 we give another characterization of the value process and use it to examine some extreme cases which admit explicit solution. Moreover, as an illustration, we also consider a diffusion market model.