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Regularly varying multivariate time series

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Abstract

Extreme values of a stationary, multivariate time series may exhibit dependence across coordinates and over time. The aim of this paper is to offer a new and potentially useful tool called tail process to describe and model such extremes. The key property is the following fact: existence of the tail process is equivalent to multivariate regular variation of finite cuts of the original process. Certain remarkable properties of the tail process are exploited to shed new light on known results on certain point processes of extremes. The theory is shown to be applicable with great ease to stationary solutions of stochastic autoregressive processes with random coefficient matrices, an interesting special case being a recently proposed factor GARCH model. In this class of models, the distribution of the tail process is calculated by a combination of analytical methods and a novel sampling algorithm.
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1. Introduction

Extreme values of a stationary, multivariate time series may exhibit dependence across coordinates and over time. The aim of this paper is to offer a new and potentially useful tool called tail process to describe and model such extremes. Let $(X_t)_{t\in\mathbb{Z}}$ be a strictly stationary time

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series of d-variate row vectors and let $\| \cdot \|$ be an arbitrary norm on $\mathbb{R}^d$. For $x > 0$, consider the process with distribution equal to

$$(\boldsymbol{X}_t/x)_{t\in\mathbb{Z}} \text{ conditionally on } \|\boldsymbol{X}_0\| > x. \tag{1.1}$$

The tail process $(\boldsymbol{Y}_t)_{t\in\mathbb{Z}}$ of $(\boldsymbol{X}_t)_{t\in\mathbb{Z}}$ is defined as the process which is the limit in law of the process in the display as $x \to \infty$, provided the limit exists and is non-degenerate. The limit is to be understood in the sense of convergence of finite-dimensional distributions. If instead we start from the process with distribution equal to

$$(\boldsymbol{X}_t/\|\boldsymbol{X}_0\|)_{t\in\mathbb{Z}} \text{ conditionally on } \|\boldsymbol{X}_0\| > x,$$

then the limit in law is denoted by $(\boldsymbol{\Theta}_t)_{t\in\mathbb{Z}}$ and is called the spectral tail process or spectral process in short. By the continuous mapping theorem, the processes $(\boldsymbol{\Theta}_t)_{t\in\mathbb{Z}}$ and $(\boldsymbol{Y}_t/\|\boldsymbol{Y}_0\|)_{t\in\mathbb{Z}}$ are equal in law.

The key property to justify these new concepts is the following fact: the tail process of a stationary process $(\boldsymbol{X}_t)_{t\in\mathbb{Z}}$ exists if and only if $(\boldsymbol{X}_t)_{t\in\mathbb{Z}}$ is jointly regularly varying, that is, if all vectors of the form $(\boldsymbol{X}_k, \dots, \boldsymbol{X}_l)$ are multivariate regularly varying. In that case, the law of $\boldsymbol{\Theta}_0$ is just the spectral measure of $\boldsymbol{X}_0$. Multivariate regular variation is a property shared by many interesting processes: linear processes with heavy-tailed innovations [28,15], stationary solutions to stochastic recurrence equations with ARCH(1) and GARCH(1, 1) processes as special cases [17,2], and moving maxima processes with heavy-tailed innovations [7,8,35]. Recall that the law of a d-dimensional random vector $\boldsymbol{X}$ is multivariate regularly varying with index $\alpha \in (0, \infty)$ if for some norm $\| \cdot \|$ on $\mathbb{R}^d$ there exists a random vector $\boldsymbol{\Theta}$ on the unit sphere $\mathbb{S}^{d-1} = \{\boldsymbol{x} \in \mathbb{R}^d \mid \|\boldsymbol{x}\| = 1\}$ such that for every $u \in (0, \infty)$ and as $x \to \infty$,

$$\frac{1}{\Pr(\|\boldsymbol{X}\| > x)} \Pr(\|\boldsymbol{X}\| > ux, \boldsymbol{X}/\|\boldsymbol{X}\| \in \cdot\,) \xrightarrow{w} u^{-\alpha} \Pr(\boldsymbol{\Theta} \in \cdot\,), \tag{1.2}$$

where $\xrightarrow{w}$ denotes weak convergence. The law of $\boldsymbol{\Theta}$ is called the spectral measure of $\boldsymbol{X}$. The definition of regular variation does not depend on the particular norm chosen in the sense that (1.2) holds for some norm if and only if it holds for every norm, the spectral measure of course depending on the norm. For more background on multivariate regular variation, see for instance [26,27,21,1,14].

The tail process $(\boldsymbol{Y}_t)_{t\in\mathbb{Z}}$ is itself not stationary, but a recurrent theme in this paper is the fact that the distribution of the tail process $(\boldsymbol{Y}_t)_{t\in\mathbb{Z}}$ is determined by the distribution of the forward tail process $(\boldsymbol{Y}_t)_{t\geqslant 0}$. In particular, it is sufficient to verify convergence of the finite-dimensional distributions of the process in (1.1) for non-negative t only. In other words, it suffices to look at the present and the future of the process given that it starts in a value far away from the origin. This property is particularly useful for processes with a Markovian structure, and we will exploit it to find the tail process of the multivariate stationary autoregressive processes with random coefficients.

Point processes form a natural language to formulate limit theorems concerning extremes of stationary time series. One such process, studied in [5,6], is

$$N_n = \sum_{t=1}^{n} \delta_{a_n^{-1}\boldsymbol{X}_t},$$

where $\delta_{\boldsymbol{x}}$ denotes the Dirac measure at $\boldsymbol{x}$ and where $(a_n)_{n\geqslant 1}$ is a positive real sequence such that $n \Pr(\|\boldsymbol{X}_0\| > a_n) \to 1$ as $n \to \infty$. The point process N_n is instrumental for instance in

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