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# Optimal inventory control with path-dependent cost criteria

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**Abstract**

This paper deals with a stochastic control problem arising from inventory control, in which the cost structure depends on the current position as well as the running maximum of the state process. A control mechanism is introduced to control the growth of the running maximum which represents the required storage capacity. The infinite horizon discounted cost minimization problem is addressed and it is used to derive a complete solution to the long-run average cost minimization problem. An associated control cost minimization problem subject to a storage capacity constraint is also addressed. Finally, as an application of the above results, a related infinite-horizon discounted control problem with a regime-switching inventory model is also solved.


*MSC:* 93E20; 60H30

*Keywords:* Inventory control; Running maximum; Ergodic control; Constrained minimization; Regime-switching diffusions

## 1. Introduction

In this article, we address a stochastic control problem motivated by inventory control subject to capacity expansion. Consider an inventory model for a product where the *inventory netflow*

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fluctuates as a random process. Here the netflow process represents the difference between the supplies of the product to the storage facility and customer demands. A negative netflow represents the inventory backlog at the time. A storage capacity is used to store the product when the inventory level is positive. However, rapid expansion of the storage capacity is expensive and therefore a control is enforced to the netflow process in order to maintain a gradual capacity expansion. In practice, exercising the control amounts to providing customer sales discounts for the product. We introduce a cost structure associated with the controlled netflow process and it consists of two components: *a control cost* to represent the loss of profits due to the exercise of control and *a capacity expansion cost* that is proportional to the storage expansion.

Stochastic control problems motivated by inventory models and stochastic processing systems have a long history and is a common theme in the literature. Some early work can be found in [2,17,16,33] and the references therein. More recent work include [9,11,10,12,5,6,3,4,26], to name just a few. In [4], a discrete inventory system is studied, where the unmet demand is lost and the excess inventory is subject to shrinkage. In the recent work of [5,9,11,10,26,33], the inventory processes are modeled by drifted Brownian motions with or without Poisson demand or more general jump diffusions. The adjustments to inventory levels are represented by controlled impulse jumps (upward or downward) and each such impulse jump includes a fixed cost and a proportional cost. In addition, a state-dependent holding and/or backorder cost is also included. In these articles, they address the optimal inventory control problem under the long-run average or discounted cost minimization criteria. Often, the focus is to establish the optimality of an $(s, S)$-policy. The article of [12] considers a single inventory model subject to independent stochastic demand and item returns. Using an appropriate transformation, they derive the optimality of an $(s, S)$-policy under the average cost minimization criterion.

This work takes a different perspective toward inventory control problems. We begin with a netflow process modeled by a drifted Brownian motion process. Next, control policies are introduced in order to avoid costly rapid capacity expansion and they represent discount sales or discarding items in the context of a perishable inventory. But exercising such controls may result in loss of profits. Therefore, it is desirable to determine a control policy that minimizes a cost structure which constitutes control costs and capacity expansion costs. To this end, we first introduce an infinite horizon discounted cost functional $J$ in Eq. (2.5), with a convex control cost and a linear capacity expansion cost.

It is natural to use the running maximum process of the netflow to model the storage expansion. Since the running maximum at any time $t \geq 0$ depends on the entire history of the underlying netflow process in the interval $[0, t]$, it is of path-dependent nature. With such a path-dependent term in the cost functional $J$ of (2.5), the analysis of the problem is nontrivial. The usual methodology of stochastic control theory (such as [13,25,38]) is not applicable directly. Indeed, a heuristic application of the usual dynamical programming principle yields a Hamilton–Jacobi–Bellman (HJB) equation of an enlarged dimension; which does not lead to the value function and optimal control policies. The paper [18] deals with a stochastic control problem involving a running maximum process. Thanks to the special features of their cost structure, they were able to approach the problem by first solving a family of auxiliary regular stochastic control problems. This methodology is further developed in [19,20] to deal with optimal control and replacement problem with state-dependent failure rate. On the other hand, the literature on Russian options, e.g., [29,30] and the references therein, deal with exotic financial options that involve the running maximum process of the underlying stock process. In these works, the value function and optimal stopping policies were obtained through appropriate transformations.