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Arbitrage of the first kind and filtration enlargements in semimartingale financial models

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Abstract

In a general semimartingale financial model, we study the stability of the *No Arbitrage of the First Kind* (NA_1) (or, equivalently, *No Unbounded Profit with Bounded Risk*) condition under initial and under progressive filtration enlargements. In both cases, we provide a simple and general condition which is sufficient to ensure this stability for *any fixed* semimartingale model. Furthermore, we give a characterisation of the NA_1 stability for *all* semimartingale models.

MSC: 60G44; 91G10

Keywords: Progressive enlargement of filtrations; Initial enlargement of filtrations; Arbitrage of the first kind; Martingale deflator

0. Introduction

In financial mathematics, market models with different sets of information have been widely studied, especially in relation to insider trading and credit risk modelling (see e.g. [22] and the references therein). Typically, one starts by postulating a model with respect to a given information set and then enlarges that set with some additional information not originally present

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in the market. From a mathematical point of view, this corresponds to considering an *enlargement* of the original filtration on a given filtered probability space. Since the model aims at representing a financial market, a fundamental question is whether the additional information allows for arbitrage profits.

The present paper aims at answering the above question in the context of models driven by general semimartingales, both in the case where the additional information is added in a progressive way over time, and in the case where the additional information is fully added at the initial time. Referring to the terminology of the theory of enlargement of filtrations (see [19] for a complete account of the theory and [22, Section 5.9] and [31, Ch. VI] for a presentation of the main results), this corresponds to considering a filtration obtained as a *progressive* or as an *initial* enlargement, respectively, of the original filtration.

Our analysis focuses on the *No Arbitrage of the First Kind* (NA_1) condition (see [24]), which is equivalent to the *No Unbounded Profit with Bounded Risk* (NUPBR) condition (see [24, Proposition 1]). Mathematically, condition NA_1 is equivalent to existence of strictly positive local martingale deflators, and can be shown to be the minimal condition ensuring the well-posedness of expected utility maximisation problems (see [28, Proposition 4.19]). In the case of a progressive enlargement with respect to a random time τ, we study the stability of NA_1 on the random time horizon $[0, \tau]$, showing that the existence of arbitrages of the first kind in the enlarged filtration is crucially linked to the possibility of the asset-price process exhibiting a jump at the same time when a particular nonnegative local martingale in the original filtration jumps to zero. In turn, we show that the possibility of the latter event is intimately related to how local martingales from the original filtration behave in the enlarged filtration, up to a suitable normalisation. In the case of an initial enlargement of the original filtration, and under the classical density hypothesis of [18], we establish an analogous set of results, showing that the validity of NA_1 in the enlarged filtration is linked to the possibility of the asset-price process jumping at the same time when a family of nonnegative martingales in the original filtration jumps to zero. In turn, as in the case of progressive enlargements, the latter possibility also fully characterises how local martingales from the original filtration behave in the enlarged filtration, up to a suitable normalisation.

In both cases of progressive and of initial enlargement, these results allow us to provide an easy sufficient condition ensuring the NA_1 stability for *a fixed* semimartingale model, as well as to explicitly characterise the stability of NA_1 for *all* semimartingale models. Although absent in the statements of our main results, an inspection of their proofs reveals a hands-on approach to the problem: using local martingale deflators in the original filtration, we explicitly construct local martingale deflators in the enlarged filtration in order to show validity of condition NA_1. In the process, we obtain some interesting new results on progressive as well as initial filtration enlargement, showing how the super/local martingale property of a process can be transferred from the original filtration to the enlarged one by suitably deflating the process.

For progressive filtration enlargement with respect to an honest time τ (see [31, Ch. VI]), examples of arbitrage profits are provided in [15,35,8]. In the context of continuous semimartingale models, as shown in [8, Theorem 4.1] (see also [29, Lemma 6.7]), condition NA_1 is always valid in the enlarged filtration on the random time horizon $[0, \tau]$. In the case of general semimartingale models, this is no longer true, see the example in Section 1.5.1. In that context, the recent paper [1] addresses the issue of NA_1 stability in progressively enlarged filtrations and represents one of the sources of inspiration for the present work. In particular, the key role of conditions equivalent to those given in Theorem 1.4 and Remark 1.5 has been first pointed out and proved in [1] (see Remark 1.6) and the characterisation we obtained in