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Stochastic Processes and their Applications 126 (2016) 1785–1818

stochastic processes and their applications

www.elsevier.com/locate/spa

Large deviations for Markov-modulated diffusion processes with rapid switching

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Received 12 January 2015; received in revised form 9 December 2015; accepted 12 December 2015
Available online 18 December 2015

Abstract

In this paper, we study small noise asymptotics of Markov-modulated diffusion processes in the regime that the modulating Markov chain is rapidly switching. We prove the joint sample-path large deviations principle for the Markov-modulated diffusion process and the occupation measure of the Markov chain (which evidently also yields the large deviations principle for each of them separately by applying the contraction principle). The structure of the proof is such that we first prove exponential tightness, and then establish a local large deviations principle (where the latter part is split into proving the corresponding upper bound and lower bound).

Keywords: Diffusion processes; Markov modulation; Large deviations; Stochastic exponentials; Occupation measure

1. Introduction

The setting studied in this paper is the following. We consider a complete probability space $(\Omega, \mathscr{F}, \mathbb{P})$ with a filtration $\{\mathscr{F}_t\}_{t\in\mathbb{R}_+}$, where $\mathbb{R}_+ := [0, +\infty)$. $\mathscr{F}_0$ contains all the $\mathbb{P}$-null sets of $\mathscr{F}$, and $\{\mathscr{F}_t\}_{t\in\mathbb{R}_+}$ is right continuous. Let X_t be a finite-state time-homogeneous Markov

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http://dx.doi.org/10.1016/j.spa.2015.12.005
0304-4149/

chain with transition intensity matrix Q and state space $\mathbb{S} := \{1, \dots, d\}$ for some $d \in \mathbb{N}$. The Markov-modulated diffusion process is defined as the unique solution to

$$M_t = M_0 + \int_0^t b(X_s, M_s)\mathrm{d}s + \int_0^t \sigma(X_s, M_s)\mathrm{d}B_s,$$

where B_t is a standard Brownian motion. We assume that there exist i, x such that $\sigma(i, x) \neq 0$ throughout this paper. The concept of Markov modulation is also known as 'regime switching'; the Markov chain X_t is often referred to as the 'background process', or the 'modulating Markov chain'.

The objective of this paper is to study the above stochastic differential equation under a particular parameter scaling. For a strictly positive (but typically small) ϵ, we scale Q to $Q/\epsilon =: Q^\epsilon$, and denote by X_t^ϵ the Markov chain with this transition intensity matrix Q^ϵ. If the expected number of jumps per unit time is y for X_t, then the time-scaling entails that it is y/ϵ for X_t^ϵ. One could therefore say that the Markov chain has been sped up by a factor ϵ^{-1}, and, as a consequence, X_t^ϵ switches rapidly among its states when ϵ is small. A classical topic in large deviations theory, initiated by Freidlin and Wentzell [9], concerns small-noise large deviations. In this paper, we investigate how rapid-switching behavior of X_t^ϵ affects the small-noise asymptotics of X_t^ϵ-modulated diffusion processes on the interval $[0, T]$ (for any fixed strictly positive T).

Let us make the scaling regime considered more concrete now. Importantly, it concerns a scaling of the function $\sigma(\cdot, \cdot)$ to $\sqrt{\epsilon}\sigma(\cdot, \cdot)$ in the Markov-modulated diffusion, but at the same time we speed up the Markovian background process in the way we described above. The resulting process M_t^ϵ is defined as the unique strong solution to

$$M_t^\epsilon = M_0^\epsilon + \int_0^t b(X_s^\epsilon, M_s^\epsilon)\mathrm{d}s + \sqrt{\epsilon}\int_0^t \sigma(X_s^\epsilon, M_s^\epsilon)\mathrm{d}B_s, \tag{1}$$

where we recall that X_t^ϵ has transition intensity matrix Q^ϵ. Focusing on the regime that $\epsilon \to 0$, we call in the sequel M_t^ϵ the Markov-modulated diffusion process with rapid switching. For simplicity, we will assume throughout this paper that $M_0^\epsilon \equiv 0$, whereas X_0^ϵ starts at an arbitrary $x \in \mathbb{S}$, for all ϵ. When we write e.g. $\mathbb{E}[M_t^\epsilon]$, this is to be understood as the expectation of M_t^ϵ with the above initial conditions.

Since M_t^ϵ evolves in the random environment of X_t^ϵ, we need to separate the effects of the vanishing of the diffusion term and the fast varying of the Markov chain, but at the same time to keep track of both of them. Since the scaling Q to Q/ϵ is equivalent to speeding up time by a factor ϵ^{-1}, one could informally say that X_t^ϵ relates to a faster time scale than M_t^ϵ, and therefore essentially exhibits stationary behavior 'around' this specific t. Then it is custom to consider the occupation measure of X_t^ϵ, which is defined on $\Omega \times [0, T] \times \mathbb{S}$ as

$$\nu^\epsilon(\omega; t, i) = \int_0^t \mathbf{1}_{\{X_s^\epsilon(\omega)=i\}}\mathrm{d}s. \tag{2}$$

As its name suggests, $\nu^\epsilon(\cdot; T, i)$ measures the time X_t^ϵ spends in state i during the time interval $[0, T]$. Moreover, we can use the derivative of $\nu^\epsilon(t)$ to gauge the infinitesimal change of the occupation measure of X_t^ϵ, at any $t \in [0, T]$. Henceforth we will thus investigate the joint process $(M^\epsilon, \nu^\epsilon)$, the main object studied in this paper.

A celebrated result in Donsker and Varadhan [6] concerns the large deviations principle (LDP) for $\nu^1(\omega; t, \cdot)/t$ as $t \to \infty$ (i.e., the LDP of the fraction of time spent in the individual states of the background process). The setting of the present paper, however, involves the *sample-*