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A limit theorem for the time of ruin in a Gaussian ruin problem

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Abstract

For certain Gaussian processes $X(t)$ with trend $-ct^\beta$ and variance $V^2(t)$, the ruin time is analyzed where the ruin time is defined as the first time point t such that $X(t) - ct^\beta \geq u$. The ruin time is of interest in finance and actuarial subjects. But the ruin time is also of interest in other applications, e.g. in telecommunications where it indicates the first time of an overflow. We derive the asymptotic distribution of the ruin time as $u \to \infty$ showing that the limiting distribution depends on the parameters β, $V(t)$ and the correlation function of $X(t)$.

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1. Introduction

Let $X(t), t \geq 0$, be a Gaussian process with mean 0 and variance $V^2(t)$, assuming that $V^2(t)$ is regularly varying at infinity with index $2H, 0 < H < 1$. Let the trajectories of X be a.s. continuous and $X(0) = 0$ a.s. Take $\beta > H$ and $c > 0$. In [9] we considered under additional

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restrictions the probability of ruin

$$P\{\sup_{t\geq 0}(X(t) - ct^{\beta}) > u\}. \tag{1}$$

For $u \to \infty$, the limiting probability of (1) is derived for a certain class of Gaussian processes which includes self-similar Gaussian processes and fractional Brownian motions. Other cases are discussed for instance in [6,11] (see also references in this paper).

In recent years, research in ruin theory has focused also on properties of distribution of the time to ruin. See for example [3,5,7,15] for classical risk models as well as risk models perturbed by diffusion. The aim of this note is to demonstrate that asymptotic methods in the theory of Gaussian processes allow us to get approximations not only for the ruin probability (1) but also for the time of ruin,

$$\tau_u = \inf\{t \geq 0 : u + ct^{\beta} - X(t) \leq 0\} \leq \infty.$$

If ruin happens, then one wants to know when it happens. Hence, continuing the considerations of Hüsler and Piterbarg [9], we prove a rather general conditional limit theorem for τ_u as $u \to \infty$, given that the ruin occurs, i.e. $\tau_u < \infty$.

In other contexts, e.g. in telecommunications, such models $X(t) - ct^{\beta}$ are considered for the storage at time t. Hence the ruin time indicates the first time of an overflow with storage space u.

2. Main result

As discussed in [9], it is more convenient to study the family of zero-mean Gaussian processes

$$X^{(u)}(s) = \frac{X(su^{1/\beta})}{V(u^{1/\beta})(1 + cs^{\beta})}, \quad s > 0.$$

By changing time $t = su^{1/\beta}$, we have

$$P\left\{\sup_{t\geq 0}(X(t) - ct^{\beta}) > u\right\} = P\left\{\sup_{s\geq 0} X^{(u)}(s) > \frac{u}{V(u^{1/\beta})}\right\},$$

and $\tau_u = u^{1/\beta}\tau$, where

$$\tau := \inf\left\{s \geq 0 : \frac{u}{V(u^{1/\beta})} - X^{(u)}(s) \leq 0\right\}, \tag{2}$$

i.e. τ denotes the ruin time in the transformed time. The process $X^{(u)}(s)$ with mean zero is not standardized; its variance equals $v_u^{-2}(s)$, where

$$v_u(s) = \frac{s^H V(u^{1/\beta})}{V(su^{1/\beta})} v(s) \quad \text{with } v(s) = s^{-H} + cs^{\beta - H},$$

and by assumption,

$$\frac{s^H V(u^{1/\beta})}{V(su^{1/\beta})} \to 1 \tag{3}$$

as $u \to \infty$, uniformly in s in any bounded interval not containing 0. We need a stronger assumption on V; see assumption A1 below. This condition is similar to other second-order