Available online at www.sciencedirect.com
ScienceDirect
Stochastic Processes and their Applications 117 (2007) 596–612
stochastic processes and their applications
www.elsevier.com/locate/spa

Large deviations and phase transition for random walks in random nonnegative potentials

Markus Flury*

Universität Zürich, Institut für Mathematik, Winterthurerstr. 190, CH-8057 Zürich, Switzerland

Received 13 December 2005; received in revised form 29 August 2006; accepted 18 September 2006
Available online 9 October 2006

Abstract

We establish large deviation principles and phase transition results for both quenched and annealed settings of nearest-neighbor random walks with constant drift in random nonnegative potentials on $\mathbb{Z}^d$. We complement the analysis of M.P.W. Zerner [Directional decay of the Green's function for a random nonnegative potential on $\mathbb{Z}^d$, Ann. Appl. Probab. 8 (1996) 246–280], where a shape theorem on the Lyapunov functions and a large deviation principle in absence of the drift are achieved for the quenched setting.

Keywords: Random walk; Random potential; Path measure; Lyapunov function; Shape theorem; Large deviation principle; Phase transition

1. Introduction

Let $\mathcal{S} = (S(n))_{n\in\mathbb{N}_0}$ be a symmetric nearest-neighbor random walk on $\mathbb{Z}^d$ starting at the origin, and denote by P, respectively E, the associated probability measure, respectively expectation. The aim of this article is a probabilistic description of the long-time behavior of the random walk, endowed with a drift and evolving in a random environment given by a random potential on the lattice. This description will be done for concrete realizations of the environment, the *quenched* setting, as well as for the averaged environment, the so-called *annealed* setting. For details, we make the following assumptions:

(Qu) $\mathbb{V} = (V_x)_{x\in\mathbb{Z}^d}$ is a family of independent, identically and not trivially distributed random variables in $L^d(\Omega, \mathcal{F}, \mathbb{P})$, which is independent of the random walk itself and satisfies $\operatorname{ess\,inf} V_x = 0$.

* Tel.: +41 4463 55843; fax: +41 4463 55705.
E-mail address: mflury@amath.unizh.ch.

0304-4149/$ - see front matter
doi:10.1016/j.spa.2006.09.006

(An) $\varphi : [0, \infty) \to [0, \infty)$ is a non constant, non decreasing and concave function with $\varphi(0) = 0$ and $\lim_{t\to\infty} \varphi(t)/t = 0$.

For $\omega \in \Omega$, $n \in \mathbb{N}$ and $h \in \mathbb{R}^d$, the *quenched path measure* $Q^h_{n,\omega}$ for the random walk $\mathcal{S}$ with constant drift h under the path potential

$$\Psi(n, \omega) \stackrel{\text{def}}{=} \sum_{1\le m\le n} V_{S(m)}(\omega)$$

is defined by means of the density function

$$\frac{\mathrm{d}Q^h_{n,\omega}}{\mathrm{d}P} \stackrel{\text{def}}{=} \frac{1}{Z^h_{n,\omega}} \exp(h \cdot S(n) - \Psi(n, \omega)),$$

where $Z^h_{n,\omega}$ denotes the corresponding (quenched) normalization. Notice that $Q^h_{n,\cdot}$ is a random probability measure, the randomness coming from the random potential $\Psi(n, \cdot)$.

For $x \in \mathbb{Z}^d$, let now

$$l_x(n) \stackrel{\text{def}}{=} \sharp\{1 \le m \le n : \ S(m) = x\}$$

denote the number of the random walk's visits to the site x up to time n. The *annealed path measure* Q^h_n for the random walk $\mathcal{S}$ with constant drift h under the path potential

$$\Phi(n) \stackrel{\text{def}}{=} \sum_{x\in\mathbb{Z}^d} \varphi(l_x(n))$$

is defined by means of the density function

$$\frac{\mathrm{d}Q^h_n}{\mathrm{d}P} \stackrel{\text{def}}{=} \frac{1}{Z^h_n} \exp(h \cdot S(n) - \Phi(n)),$$

where Z^h_n is the corresponding (annealed) normalization constant.

The model we come to introduce is a discrete-setting model for a particle moving in a random media. In the quenched setting, the walker jumps from site to site, thereby trying to avoid those regions where the potential takes on high values. The drift however implies a restriction in the search of such an "optimal strategy" by imposing a particular direction to the walk.

We shall point out that in the definition of the annealed path measures we are making a slight abuse of standard terminology. To clarify this aspect, consider

$$\varphi_{\mathbb{V}}(t) \stackrel{\text{def}}{=} -\log \mathbb{E} \exp(-t V_x), \quad t \in [0, \infty),$$

for a given potential $\mathbb{V}$. By Hölder inequality, dominated convergence and the assumption $\operatorname{ess\,inf} V_x = 0$, it is easy to see that $\varphi_{\mathbb{V}}$ fulfills the requirements (An). Let $Q^h_{\mathbb{V},n}$ denote the annealed path measure corresponding to $\varphi_{\mathbb{V}}$. The quenched potential can be rewritten as

$$\Psi(n, \omega) = \sum_{x\in\mathbb{Z}^d} l_x(n) V_x(\omega).$$

By the independence assumption on the potential, it now is easily seen that

$$\frac{\mathrm{d}Q^h_{\mathbb{V},n}}{\mathrm{d}P} = \frac{1}{\mathbb{E}Z^h_{n,\cdot}} \mathbb{E}\left[\exp(h \cdot S(n) - \Psi(n, \cdot))\right]$$