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Robust superhedging with jumps and diffusion

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Abstract

We establish a nondominated version of the optional decomposition theorem in a setting that includes jump processes with nonvanishing diffusion as well as general continuous processes. This result is used to derive a robust superhedging duality and the existence of an optimal superhedging strategy for general contingent claims. We illustrate the main results in the framework of nonlinear Lévy processes.

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1. Introduction

The classical optional decomposition theorem states that given a process Y which is a supermartingale under all equivalent martingale measures of some reference process S, there exists an integrand H such that $Y - H \bullet S$ is nonincreasing, where $H \bullet S$ denotes the stochastic integral. Stated differently, Y admits the decomposition $Y = Y_0 + H \bullet S - K$ for some nondecreasing process K. This result is stated on a given probability space $(\Omega, \mathcal{F}, P_*)$; without loss of generality, one can assume that S is itself a P_* martingale. The optional decomposition theorem is due to [15] in the case of a continuous process S, while the case with jumps is due to [21] under a boundedness assumption and [16] in the general case. An alternative proof was presented in [8], and [17] extended the result to include portfolio constraints. Optional decomposition theorems are important for applications in mathematical finance, in particular for superreplication pricing and portfolio optimization.

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The first result of this paper (Theorem 2.4) is a version of the optional decomposition theorem which is suitable in the context of model uncertainty: it does not require a reference measure. More precisely, we consider a set $\mathfrak{P}$ of probabilities, possibly nondominated in the sense that its elements are not dominated by a single reference probability P_*. Suppose that S is a càdlàg local martingale under all elements of $\mathfrak{P}$, and that $\mathfrak{P}$ contains all equivalent local martingale measures of its elements. If Y is a càdlàg supermartingale under all $P \in \mathfrak{P}$, we show that there exists an integrand H such that $Y - H \bullet S$ is nonincreasing P-a.s. for all $P \in \mathfrak{P}$. This result is obtained under a technical condition that we call dominating diffusion property (Definition 2.2): for all $P \in \mathfrak{P}$, the jump characteristic ν^P of S is dominated by the diffusion characteristic C^P. This includes the case where S is an Itô semimartingale with jumps and nonvanishing diffusion, as well as the case of a general continuous process. The dominating diffusion property allows us to define H in terms of the joint diffusion characteristic of (S, Y) and thus to take advantage of the fact that the latter can be constructed in an aggregated way (i.e., simultaneously for all P). The proof that this strategy is superreplicating capitalizes on the classical optional decomposition theorem under each $P \in \mathfrak{P}$. Thus, the argument is quite simple, with the advantage that the result is general and the proof is versatile (see [5] for an adaptation to a different setting). In particular, we do not require delicate separability conditions on the filtration or compactness assumptions on $\mathfrak{P}$.

The second result (Theorem 3.2) is a superhedging duality in the setting of model uncertainty. In a setup where Skorohod space is used as the underlying measurable space and the set $\mathfrak{P}$ satisfies certain dynamic programming conditions, it is shown that given a measurable function f at the time horizon T, the robust superhedging price

$$\pi(f) := \inf\{x \in \mathbb{R} : \exists H \text{ with } x + H \bullet S_T \geq f \;\; P\text{-a.s. for all } P \in \mathfrak{P}\}$$

satisfies the duality relation

$$\pi(f) = \sup_{P \in \mathfrak{P}} E^P[f].$$

Moreover, the infimum is attained; i.e., an optimal superhedging strategy exists. We construct this strategy through the above optional decomposition result.

Finally, we illustrate our main results in the setting of nonlinear Lévy processes; this is a natural example where the set $\mathfrak{P}$ is defined in terms of the characteristics of S. We characterize the conditions of our main results in terms of the model primitives and discuss further aspects of our problem formulation.

The main novelty in our results is the applicability to nondominated continuous-time models with jumps. To the best of our knowledge, there is no extant result providing the existence of an optimal strategy or an optional decomposition theorem in this framework. The only previous result is the duality statement of [13] in the context of optimal transport; there, the absence of a duality gap is proved by a weak approximation with discrete models and the superreplication is formulated in a pathwise fashion. In ongoing independent work [7], absence of a duality gap will be established by functional analytic methods, though under a compactness condition in Skorohod space which is generally not satisfied in our setting.

The case of continuous processes (i.e., volatility uncertainty) is better studied. The duality formula in this context is investigated by [10] from a capacity-theoretic point of view, [34,36,38] use an approximation by Markovian control problems, and [12] uses a weak approximation based on dominated models. On the other hand, [22,29,31,35,37] use an aggregation argument which can be seen as a predecessor of our proof; however, they rely on the Doob–Meyer decomposition theorem (under each $P \in \mathfrak{P}$) and this forces them to assume that each $P \in \mathfrak{P}$ corresponds to a