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Pruning of CRT-sub-trees

Romain Abraham[a], Jean-François Delmas[b], Hui He[c,*]

[a] *MAPMO, CNRS UMR 6628, Fédération Denis Poisson FR 2964, Université d'Orléans, B.P. 6759, 45067 Orléans cedex 2, France*

[b] *CERMICS, Université Paris-Est, 6-8 av. Blaise Pascal, Champs-sur-Marne, 77455 Marne La Vallée, France*

[c] *Laboratory of Mathematics and Complex Systems, School of Mathematical Sciences, Beijing Normal University, Beijing 100875, PR China*

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Abstract

We study the pruning process developed by Abraham and Delmas (2012) on the discrete Galton–Watson sub-trees of the Lévy tree which are obtained by considering the minimal sub-tree connecting the root and leaves chosen uniformly at rate λ, see Duquesne and Le Gall (2002). The tree-valued process, as λ increases, has been studied by Duquesne and Winkel (2007). Notice that we have a tree-valued process indexed by two parameters: the pruning parameter θ and the intensity λ. Our main results are: construction and marginals of the pruning process, representation of the pruning process (forward in time that is as θ increases) and description of the growing process (backward in time that is as θ decreases) and distribution of the ascension time (or explosion time of the backward process) as well as the tree at the ascension time. A by-product of our result is that the super-critical Lévy trees independently introduced by Abraham and Delmas (2012) and Duquesne and Winkel (2007) coincide. This work is also related to the pruning of discrete Galton–Watson trees studied by Abraham, Delmas and He (2012).

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Keywords: Pruning; Branching process; Galton–Watson process; Random tree; CRT; Tree-valued process; Girsanov transformation

* Corresponding author.
E-mail addresses: romain.abraham@univ-orleans.fr (R. Abraham), delmas@cermics.enpc.fr (J.-F. Delmas), hehui@bnu.edu.cn (H. He).

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1. Introduction

The study of pruning of Galton–Watson trees has been initiated by Aldous and Pitman [9]. Roughly speaking, it corresponds to the percolation on edges: an edge is uniformly chosen at random in a Galton–Watson tree and is removed, and the connected component containing the root is kept. This procedure is then iterated. This process can be extended backward in time. It corresponds then to a non-decreasing tree-valued process. The ascension time A is then the first time at which this tree-valued process reaches an unbounded tree. In [9], the authors give the joint distribution of A as well as the tree just before the ascension time (in backward time).

The limits of Galton–Watson trees are the so called continuum Lévy trees, see [6,7,20,13]; they are characterized by a branching mechanism ψ which is also a Lévy exponent. The result for the pruning process on Galton–Watson trees was then extended by Abraham and Delmas [1] to a process indexed by time θ whose marginals are continuum Lévy trees. In the setting of the Brownian continuum random tree, which corresponds to a quadratic branching mechanism, the pruning procedure is uniform on the skeleton, see also Aldous and Pitman [8] for a fragmentation point of view in this case. This is the analogue of [9]. However in the general Lévy case, one has to take into account the pruning of nodes with a rate given by their "size" or "mass", which is defined as the asymptotic number of small trees attached to the node. This result in the continuous setting motivated a new pruning procedure on the nodes of Galton–Watson trees, which was developed by Abraham, Delmas and He [2]. In this case, the pruning happens on the nodes with rate depending on the degree of the nodes.

In the present work, we study the pruning process developed in [1] on the discrete Galton–Watson sub-trees of the Lévy tree. The discrete Galton–Watson sub-trees of the Lévy trees are obtained by considering the minimal sub-tree connecting the root and leaves chosen uniformly with rate $\lambda \geq 0$, see Duquesne and Le Gall [13]. The tree-valued process, as λ increases, has been studied by Duquesne and Winkel [14], in particular to construct super-critical Lévy trees. Notice that super-critical Lévy trees have also been defined in [1]. One of the by-product of our results is that the two definitions coincide, see Section 5. Notice that we have a tree-valued process indexed by two parameters θ (as in [9,1]) and λ (as in [14]). The other main results are: construction and marginals of the pruning process in Section 4, representation of the pruning process (forward in time that is as θ increases) and description of the growing process (backward in time that is as θ decreases) in Section 6, some remarks on martingales related to the number of leaves in Section 7, distributions of the ascension time and of the tree at the ascension time in Section 8.

Now, we present more precisely our results. Let ψ be a branching mechanism satisfying the Grey condition (see (6) in Section 2.6). A priori, the branching mechanism ψ is defined on $[0, +\infty)$ but we may extend it on a part of $(-\infty, 0)$ using formula (5). For every θ such that $\psi(\theta)$ exists, we define the branching mechanism ψ_θ by:

$$\psi_\theta(q) = \psi(q + \theta) - \psi(\theta) \quad \text{for all } q \geq 0,$$

and denote by Θ the set of θ for which $\psi(\theta)$ exists. Note that ψ_θ satisfies the Grey condition (6). We consider the tree-valued process $(\mathcal{T}_\theta, \theta \in \Theta)$ introduced in [1], corresponding to a uniform pruning on the skeleton and to a pruning at nodes with rate depending on its size. We recall that $\mathcal{T}_\theta$ is a Lévy tree with branching mechanism ψ_θ. Let $\mathbf{m}^{\mathcal{T}_\theta}$ be its mass measure, which is a uniform measure on the set of leaves. Let $\tau_0(\lambda)$ be the minimal sub-tree of $\mathcal{T}_0$ generated by the root and leaves chosen before time λ according to a Poisson point measure $\mathcal{P}^0$ on $\mathbb{R}_+ \times \mathcal{T}_0$ with intensity $dt\, \mathbf{m}^{\mathcal{T}_0}$. Let M_λ be the number of chosen leaves: $M_\lambda = \mathcal{P}^0([0, \lambda] \times \mathcal{T}_0)$, so that $\tau_0(\lambda)$ is well defined for $M_\lambda \geq 1$. And we set $\tau_\theta(\lambda) = \mathcal{T}_\theta \bigcap \tau_0(\lambda)$ for $\theta \geq 0$. So we get a two-parameter