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Limit theorems for additive functionals of stationary fields, under integrability assumptions on the higher order spectral densities[☆]

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Abstract

We obtain central limit theorems for additive functionals of stationary fields under integrability conditions on the higher-order spectral densities. The proofs are based on the Hölder–Young–Brascamp–Lieb inequality.

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1. Introduction

The problem. Consider a real measurable stationary in the strict sense random field $X_t,\ t \in \mathbb{R}^d$, with $\mathbb{E}X_t = 0$, and $\mathbb{E}|X_t|^k < \infty,\ k = 2, 3, \ldots$.

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Let the random field X_t be observed over a sequence K_T of increasing dilations of a bounded convex set K of positive Lebesgue measure $|K| > 0$, containing the origin, i.e.

$$K_T = TK, \quad T \to \infty.$$

Note that $|K_T| = T^d |K|$.

We investigate the asymptotic normality of the integrals

$$S_T = \int_{t \in K_T} X_t dt \tag{1}$$

and the integrals with some weight function

$$S_T^w = \int_{t \in K_T} w(t) X_t dt \tag{2}$$

as $T \to \infty$, without imposing any extra assumption on the structure of the field such as linearity, etc.

Motivation. In the simplest cases of Gaussian or moving average processes, central limit theorems for sums (1), (2) have been often derived by the method of moments, using explicit computations involving the spectral densities.

We generalize this line of research, establishing central limit theorems for S_T and S_T^w, appropriately normalized, by the method of moments. Namely, we represent the cumulants of the integrals (1) and (2) in the spectral domain, and evaluate their asymptotic behavior using analytic tools provided by harmonic analysis. All conditions needed to prove the results will be concerned with integrability of the spectral densities of second and higher orders.

For further discussion of different approaches for derivation of CLT for stationary processes and fields, see for example [5,13,25].

Assumption A. We will assume throughout the existence of all order cumulants $c_k(t_1, t_2, \ldots, t_k) = cum_k\{X_{t_1,\ldots,}X_{t_k}\}$ for our stationary random field X_t, and also that they are representable as Fourier transforms of "cumulant spectral densities"
$f_k(\lambda_1, \ldots, \lambda_{k-1}) \in L_1(\mathbb{R}^{d(k-1)}),\ k = 2, 3, \ldots,$ i.e:

$$\begin{aligned} c_k(t_1, t_2, \ldots, t_k) &= c_k(t_1 - t_k, .., t_{k-1} - t_k, 0) \\ &= \int_{(\lambda_1,\ldots,\lambda_{k-1}) \in \mathbb{R}^{d(k-1)}} e^{i \sum_{j=1}^{k-1} \lambda_j (t_j - t_k)} f_k(\lambda_1, \ldots, \lambda_{k-1})\, d\lambda_1 \ldots d\lambda_{k-1}. \end{aligned}$$

Note: The functions $f_k(\lambda_1, \ldots, \lambda_{k-1})$ are symmetric and may be complex valued in general.

The Hölder–Young–Brascamp–Lieb (HYBL) inequality. We are able to treat here general stationary fields X_t, by making use of the powerful HYBL inequality. The computation of the cumulants of S_T (or S_T^w) in the spectral domain leads to a certain kind of convolutions of spectral densities with particular kernel functions (see formulas for the cumulants (5) and (47)). Similar convolutions have been studied in the series of papers [1,3,6,2,4,5], under the name of Fejer matroid/graph integrals.

Estimates for this kind of convolutions follow from the Hölder–Young–Brascamp–Lieb inequality which, under prescribed conditions on the integrability indices for a set of functions $f_i \in L_{p_i}(S, d\mu),\ i = 1, \ldots, n$, allows to write upper bounds for integrals of the form