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An invariance principle for stationary random fields under Hannan's condition

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Abstract

We establish an invariance principle for a general class of stationary random fields indexed by $\mathbb{Z}^d$, under Hannan's condition generalized to $\mathbb{Z}^d$. To do so we first establish a uniform integrability result for stationary orthomartingales, and second we establish a coboundary decomposition for certain stationary random fields. At last, we obtain an invariance principle by developing an orthomartingale approximation. Our invariance principle improves known results in the literature, and particularly we require only finite second moment.

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1. Introduction

Let $\{X_i\}_{i\in\mathbb{Z}^d}$ be a stationary random field with zero mean and finite variance, and let S_n be the partial sum with $n = (n_1, \dots, n_d) \in \mathbb{N}^d$

$$S_n = \sum_{\mathbf{1}\le i\le n} X_i,$$

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and we are interested in the invariance principle of normalized partial sums in form of

$$\left\{\frac{S_{\lfloor n\cdot t\rfloor}}{|n|^{1/2}}\right\}_{t\in[0,1]^d} \Rightarrow \{\sigma \mathbb{B}_t\}_{t\in[0,1]^d}, \tag{1.1}$$

where $n\cdot t = (n_1t_1, \ldots, n_dt_d)$ and $|n| = \prod_{q=1}^d n_q$. We provide a sufficient condition for the above weak convergence to hold in $D[0,1]^d$, with the limiting random field being a Brownian sheet.

The invariance principle for Brownian sheet has a long history, and people have investigated this problem from different aspects. See for example Berkes and Morrow [2], Bolthausen [3], Goldie and Morrow [13], Bradley [4] for results under mixing conditions, Basu and Dorea [1], Morkvėnas [20], Nahapetian [21], Poghosyan and Rœlly [23] for results on multiparameter martingales, and Dedecker [7,8], El Machkouri et al. [12], Wang and Woodroofe [25] for results on random fields satisfying projective-type assumptions. In particular, projective-type assumptions have been significantly developed for invariance principles for stationary sequences ($d = 1$). See for example Wu [26], Dedecker et al. [9], among others, for some recent developments. However, extending these criteria in one dimension to high dimensions is not a trivial problem.

Our goal is to establish a random-field counterpart of the invariance principle for regular stationary sequences satisfying Hannan's condition [16]. Hannan's condition consists of assuming, in dimension one,

$$\sum_{i\in\mathbb{Z}} \|P_0(X_i)\|_2 < \infty, \tag{1.2}$$

where $P_0(X_i) = \mathbb{E}(X_i \mid \mathcal{F}_0) - \mathbb{E}(X_i \mid \mathcal{F}_{-1})$ is the projection operator, with respect to certain filtration $\{\mathcal{F}_k\}_{k\in\mathbb{Z}}$ associated to the stationary sequence $\{X_k\}_{k\in\mathbb{Z}}$. Under Hannan's condition, if in addition the stationary sequence $\{X_n\}_{n\in\mathbb{N}}$ is regular (i.e. $\mathbb{E}(X_0 \mid \mathcal{F}_{-\infty}) = 0$ and X_0 is $\mathcal{F}_\infty$-measurable), then the invariance principle follows. Hannan [16] first considered the invariance principle, under the assumption that $\{X_k\}_{k\in\mathbb{Z}}$ is adapted and weakly mixing. The general case for regular sequences was established by Dedecker et al. [9, Corollary 2]. The quenched invariance principle for adapted case has been established by Cuny and Volný [6].

We first generalize Hannan's condition (1.2) to high dimension. For this purpose we need to extend the notion of the projection operator (Section 2). In particular, we focus on stationary random fields in form of

$$X_i = f \circ T_i(\{\epsilon_k\}_{k\in\mathbb{Z}^d}), \quad i \in \mathbb{Z}^d, \tag{1.3}$$

where $f : \mathbb{R}^{\mathbb{Z}^d} \to \mathbb{R}$ is a measurable function, T_i is the shift operator on $\mathbb{R}^{\mathbb{Z}^d}$ and $\{\epsilon_k\}_{k\in\mathbb{Z}^d}$ is a collection of independent and identically distributed (i.i.d.) random variables.

Our main result (Theorem 5.1) states if $\mathbb{E}X_{\mathbf{0}} = 0$, $\mathbb{E}(X_{\mathbf{0}}^2) < \infty$, and Hannan's condition holds

$$\sum_{i\in\mathbb{Z}^d} \|P_{\mathbf{0}}X_i\|_2 < \infty,$$

for some projection operator $P_{\mathbf{0}}$ to be defined (see (2.3)), then the invariance principle (1.1) holds.

We establish the invariance principle through an approximation by *orthomartingales*. As a consequence, this entails the central limit theorem in form of

$$\frac{S_n}{|n|^{1/2}} \Rightarrow \mathcal{N}(0, \sigma^2).$$