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A stochastic partially reversible investment problem on a finite time-horizon: Free-boundary analysis

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Abstract

We study a continuous-time, finite horizon, stochastic partially reversible investment problem for a firm producing a single good in a market with frictions. The production capacity is modeled as a one-dimensional, time-homogeneous, linear diffusion controlled by a bounded variation process which represents the cumulative investment–disinvestment strategy. We associate to the investment–disinvestment problem a zero-sum optimal stopping game and characterize its value function through a free-boundary problem with two moving boundaries. These are continuous, bounded and monotone curves that solve a system of non-linear integral equations of Volterra type. The optimal investment–disinvestment strategy is then shown to be a diffusion reflected at the two boundaries.

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1. Introduction

A firm represents the productive sector of a stochastic economy over a finite time-horizon and it adjusts its production capacity C making repeatedly investments and disinvestments of

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arbitrary size (we do not require the investment–disinvestment rates to be defined) at given proportional costs. Since we consider a market with frictions the firm buys and sells capacity at different fixed prices and it aims at maximizing its total net expected profit. In mathematical terms, following for instance [24], this amounts to solving the bounded variation control problem with finite horizon

$$\sup_{(\nu_+,\nu_-)} \mathbb{E}\bigg\{\int_0^T e^{-\mu_F t} R(C^{y,\nu}(t))dt - c_+ \int_0^T e^{-\mu_F t} d\nu_+(t) \\ + c_- \int_0^T e^{-\mu_F t} d\nu_-(t) + e^{-\mu_F T} G(C^{y,\nu}(T))\bigg\} \tag{1.1}$$

where the optimization is taken over all the nondecreasing processes ν_+ and ν_- representing the (cumulative) investment and disinvestment strategy, respectively. Here μ_F is the firm's manager discount factor, c_+ is the instantaneous cost of investment, c_- is the benefit from disinvestment, R the operating profit function and G a terminal gain, often referred to as a *scrap function*. We assume that the production capacity $C^{y,\nu}$ follows a stochastic, time-homogeneous, linearly controlled dynamics with $\nu := \nu_+ - \nu_-$ (cf. (2.1)).

The main goals of this papers are two. Firstly, we prove an abstract existence and uniqueness result for the optimal solution pair (ν_+^*, ν_-^*) to problem (1.1). Secondly, we provide a semi-explicit representation of such pair in terms of two continuous, bounded and monotone free-boundaries arising in a Zero-Sum Optimal Stopping Game (ZSOSG) associated to the control problem. These boundaries are characterized through a system of non-linear integral equations of Volterra type. To the best of our knowledge this is a complete novelty in the literature on bounded variation control problems with finite horizon. Moreover, we would like to remark that, differently to standard optimal stopping problems, a probabilistic analysis of time-dependent free-boundaries in ZSOSG has not received significant attention so far. A somehow related paper is the very recent work by Yam, Yung and Zhou [50] dealing with a delta-penalty game call option on a stock with a dividend payment. In that paper the optimal stopping region of a ZSOSG is analyzed in both infinite and finite time-horizon cases; authors find two optimal boundaries that uniquely solve a couple of non-linear integral equations. However, the aims of their analysis and the setting of their problem are substantially different to ours since, e.g., they do not deal with any control problem and (from a more technical point of view) they have no (unbounded) running profit. The analysis we perform on the ZSOSG builds upon the existing probabilistic theory of optimal stopping and goes beyond that extending a number of results and developing new methodologies.

Theory of investment under uncertainty has received increasing attention in the last years in Economics as well as in Mathematics (we refer for instance to Dixit and Pindyck [18] for a review). In [8,42] a firm maximizes profits over an infinite time-horizon when the operating profit function is Cobb–Douglas and depends on an exogenous stochastic shock modeled by a geometric Brownian motion. In [1,7] the authors consider the optimal investment problem under uncertainty of a firm that produces a single good with costly reversibility. The problem is formulated over an infinite time-horizon with constant returns to scale Cobb–Douglas production facing an isoelastic demand curve. In [1] the optimal investment–disinvestment policy is characterized in terms of a generalized concept of user cost of capital introduced by Jorgenson [28]. We recall that irreversible investment decisions and their timing are also related to real options as pointed out by [34,42] among others.