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Fourier transform methods for pathwise covariance estimation in the presence of jumps

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Abstract

We provide a new non-parametric Fourier procedure to estimate the trajectory of the instantaneous covariance process (from discrete observations of a multidimensional price process) in the presence of jumps extending the seminal work of Malliavin and Mancino (2002, 2009). Our approach relies on a modification of (classical) jump-robust estimators of integrated realized covariance to estimate the Fourier coefficients of the covariance trajectory. Using Fourier–Féjer inversion we reconstruct the path of the instantaneous covariance. We prove consistency and a central limit theorem (CLT) and in particular that the asymptotic estimator variance is smaller by a factor 2/3 in comparison to classical local estimators.

The procedure is robust enough to allow for an iteration and we can show theoretically and empirically how to estimate the integrated realized covariance of the instantaneous stochastic covariance process. We apply these techniques to robust calibration problems for multivariate modeling in finance, i.e., the selection of a pricing measure by using time series and derivatives' price information simultaneously.

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1. Introduction

The recent difficulties in the banking and insurance industry are to some extent due to insufficient modeling of multivariate stochastic phenomena which appear in financial markets. There are several reasons why modeling is insufficient, but the two most important ones are the following: first, realistic multivariate models are difficult to calibrate to market information due to a lack of analytic tractability, hence oversimplified models are in use in delicate multivariate situations, and, second, usually either time series data or derivatives' prices are used to select a model from a given model class but *not both* sorts of available information simultaneously. It is often argued that due to the difference of the statistical measure and the pricing measure we are actually not able to use the information simultaneously, except if we determine the statistical measure and make an ansatz for the market price of risk. Robust model calibration instead uses time series and option price information simultaneously without conjecturing about quantities which are as hard to identify as drifts.

1.1. Robust calibration

We aim to develop methods which allow for *robust calibration*, i.e., estimation and calibration of a model in a well specified sense simultaneously from time series and derivatives' prices data in order to select *a pricing measure*. Reasons why both kinds of data should enter the field of model selection in mathematical finance are the high dimensional parameter space of multivariate models and the lack of liquidly traded multi-asset options, which makes a calibration procedure solely based on derivatives' data infeasible. This difficulty can be tackled by additionally using time series of asset prices, from which certain model parameters can be determined. It is useful to demonstrate what we actually mean with *robust calibration* by means of an example: take a Heston model

$$dX_t = \kappa(\theta - X_t)dt + \sigma\sqrt{X_t}dZ_t,$$
$$dY_t = \mu - \frac{X_t}{2}dt + \sqrt{X_t}dB_t,$$

where X denotes the stochastic variance process and Y the logarithmic price of a stock. The model is written with respect to a pricing measure, i.e., $\exp(Y)$ is a martingale, if $\mu = 0$, otherwise the model is written with respect to the real world measure. Through robust calibration we have to identify the initial value X_0, Y_0, the parameters κ, θ, σ and the correlation parameter ρ between the Brownian motions Z and B in order to specify the model for purposes of pricing, hedging or short term risk management. If we specify additionally μ we can use the model for (long term) risk management.

Apparently at least the initial values X_0, Y_0, and the parameters σ and ρ do not change under equivalent measure changes, so in principal the parameters X_0, Y_0, σ, ρ *can* be identified from the observation of a single trajectory, and it does not matter with respect to which equivalent measure we observe this trajectory. On the other hand market implied values for those parameters should coincide with values estimated from the time series if the model is close to correct. Here "market implied values" means to choose model parameter values such that the model's derivatives' prices and the market prices coincide as good as possible. The other parameters κ, θ and μ can be changed under equivalent measure changes (given that we stay in the above parametrized class) and their values depend on the data, which are used to estimate them.

Formally speaking we have defined the above model on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and we consider equivalent measures $\mathbb{Q} \sim \mathbb{P}$. Having specified a set of parameters Θ and a