Available online at www.sciencedirect.com
ScienceDirect
Stochastic Processes and their Applications 124 (2014) 3807–3818
stochastic processes and their applications
www.elsevier.com/locate/spa

Singularity of full scaling limits of planar nearcritical percolation

Simon Aumann

Mathematisches Institut, Ludwig–Maximilians-Universität München, Theresienstr. 39, D-80333 München, Germany

Received 19 April 2013; received in revised form 2 July 2014; accepted 2 July 2014
Available online 8 July 2014

Abstract

We consider full scaling limits of planar nearcritical percolation in the Quad-Crossing-Topology introduced by Schramm and Smirnov. We show that two nearcritical scaling limits with different parameters are singular with respect to each other. The results hold for percolation models on rather general lattices, including bond percolation on the square lattice and site percolation on the triangular lattice.

MSC: 60K35; 82B43; 60G30; 82B27

Keywords: Nearcritical; Percolation; Full scaling limit; Singular

1. Introduction

Percolation theory has attracted more and more attention since Smirnov's proof of the conformal invariance of critical percolation interfaces on the triangular lattice. This was the missing link for the existence of a unique scaling limit of critical exploration paths. In the sequel, not only limits of exploration paths, but also limits of full percolation configurations have been explored. To obtain a scaling limit, one considers percolation on a lattice with mesh size $\eta > 0$ and lets η tend to 0. In the case of the full configuration limit, it is a-priori not clear, in what sense, or in what topology, the limit $\eta \to 0$ shall be taken. There are several possibilities, nine of them are explained in [8, p. 1770ff]. It is highly non-trivial that these different approaches yield equivalent results. Camia and Newman established the full scaling limit of critical percolation on

E-mail address: aumann@math.lmu.de.

http://dx.doi.org/10.1016/j.spa.2014.07.005
0304-4149/

the triangular lattice as an ensemble of oriented loops, see [2]. Schramm and Smirnov suggested to look at the set of quads which are crossed by the percolation configuration and constructed a nice topology for that purpose, the so-called Quad-Crossing-Topology, see [8]. Since it is closely related to the original physical motivation of percolation and it yields the existence of limit points for free (by compactness), we choose to work with Schramm and Smirnov's set-up.

They considered percolation models on tilings of the plane, rather than on lattices. Each tile is either coloured blue or yellow, independently of each other. All site or bond percolation models can be handled in this way using appropriate tilings. The results of [8] hold on a wide range of percolation models. In fact, two basic assumptions on the one-arm event and on the four-arm event are sufficient. The results of the present article also hold on rather general tilings, but a bit stronger assumptions are needed. Basically, we require the assumption of [8] on the four-arm event and the Russo–Seymour–Welsh Theory (RSW). The exact conditions are presented below. In particular, we need the arm separation lemmas of [4,5]. They should hold on any graph which is invariant under reflection in one of the coordinate axes and under rotation around the origin by an angle $\phi \in (0, \pi)$, as stated in [4, p. 112]. But the proofs are written up only for bond or site percolation on the square lattice in [4] and for site percolation on the triangular lattice in [5]. Hence we choose to formulate the exact properties we need as conditions. We will first prove our results under that conditions and we will verify them for bond percolation on the square lattice and site percolation on the triangular lattice afterwards.

We want to consider nearcritical scaling limits. Nearcritical percolation is obtained by colouring a tile blue with a probability slightly different from the critical one. The difference depends on the mesh size, but converges to zero in a well-chosen speed. It includes – for each tile – one free real parameter. The main result of the present note is the following: We consider two (inhomogeneous) nearcritical percolations such that the differences of their parameters are uniformly bounded away from zero in a macroscopic region. Then we show that any corresponding sub-sequential scaling limits are singular with respect to each other.

Nolin and Werner showed in [6, Proposition 6] that – on the triangular lattice – any (sub-sequential) scaling limit of nearcritical exploration paths is singular with respect to an SLE_6 curve, i.e. to the limit of critical exploration paths. This was extended in [1, Theorem 1], where it is shown that the limits of two nearcritical exploration paths with different parameters are singular with respect to each other. The present result is somewhat different to those results, as we will now explain. First, we consider different objects. While in [6] and [1] the singularity of exploration paths was detected, here it is the singularity of the full configurations in the Quad-Crossing-Topology. As long as the equivalence of the different descriptions of the limit object is not proven, these are independent results. In particular, it is – even on the triangular lattice – an open question, whether the exploration path as a curve is a random variable of the set of all crossed quads (cf. [3, Question 2.14]). Though the trace of the exploration path can be recovered from the set of all crossed quads, it is not clear how to detect its behaviour at double points. Thus the present result is not an easy corollary to the singularity of the exploration paths. Second, the results of [6] and [1] hold only for site percolation on the triangular lattice, whereas the results of the present article hold under rather general assumptions on the lattice, which are, for instance, also fulfilled by bond percolation on the square lattice. Last, and indeed least, the percolation may also be inhomogeneous here. Since the restriction to homogeneous percolation in [6] and [1] has only technical, but not conceptual reasons, this is only a minor difference.

The proofs use ideas from [6] and [1]. In fact, the proofs of this article are technically simpler since there is no need to consider domains with fractal boundary. In Section 2, we formally introduce the model and state all theorems and lemmas, which will be proved in Section 3.