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Large deviations for invariant measures of SPDEs with two reflecting walls

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Abstract

In this article, we establish a large deviation principle for invariant measures of solutions of stochastic partial differential equations with two reflecting walls driven by a space–time white noise.

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1. Introduction

Consider reflected stochastic partial differential equations (SPDEs) of the following type:

$$\begin{aligned} \frac{\partial u_\varepsilon(x,t)}{\partial t} &= \frac{\partial^2 u_\varepsilon(x,t)}{\partial x^2} - \alpha u_\varepsilon(x,t) + f\big(x, u_\varepsilon(x,t)\big) \\ &\quad + \varepsilon\sigma\big(x, u_\varepsilon(x,t)\big)\dot{W}(x,t) + \eta_\varepsilon(x,t) - \xi_\varepsilon(x,t) \end{aligned} \tag{1.1}$$

$$K_1(x) \le u^\varepsilon(x,t) \le K_2(x) \tag{1.2}$$

in $(x,t) \in Q := [0,1] \times \mathbb{R}_+$ while $K_1(x) \le u^\varepsilon(x,t) \le K_2(x)$. Here $\dot{W}$ is a space–time white noise. When $u_\varepsilon(x,t)$ hits $K_1(x)$ or $K_2(x)$, the additional forces are added to prevent u_ε from

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leaving $[K_1, \ K_2]$. These forces are expressed by random measures ξ_ε and η_ε in Eq. (1.1) which play a similar role as the local time in the usual Skorokhod equation constructing Brownian motions with reflecting barriers.

Parabolic SPDEs with reflection are natural extension of the widely studied deterministic parabolic obstacle problems. They also can be used to model fluctuations of an interface near a wall; see Funaki and Olla [7]. In recent years, there is a growing interest on the study of SPDEs with reflection. Several works are devoted to the existence and uniqueness of the solutions. In the case of a constant diffusion coefficient and a single reflecting barrier $K_1 = 0$, Nualart and Pardoux [8] proved the existence and uniqueness of the solutions. In the case of a non-constant diffusion coefficient and a single reflecting barrier $K_1 = 0$, the existence of a minimal solution was obtained by Donati-Martin and Pardoux [4]. The existence and particularly the uniqueness of the solutions for a fully non-linear SPDE with reflecting barrier 0 were solved by Xu and Zhang [12]. In the case of double reflecting barriers, Otobe [9] obtained the existence and uniqueness of the solutions of an SPDE driven by an additive white noise.

In addition to the existence and uniqueness, various other properties of the solution have been studied by several authors; see Donati-Martin and Pardoux [5], Zambotti [15], Dalang et al. [3] and Zhang [16].

The purpose of this paper is to establish a large deviation principle for invariant measures of the solutions of fully non-linear SPDEs with two reflecting walls (1.1). Large deviations for invariant measures of the solutions of SPDEs were previously studied in [10,2]. Our approach will be along the same lines as that in [10,2]. However, the extension is non-trivial. The extra difficulty arises from the appearance of the random measures (local times) η_ε and ξ_ε in Eq. (1.1). We need to carefully analyze the local time terms in the skeleton equations and provide some uniform estimates for the penalized approximating equations.

The rest of the paper is organized as follows. In Section 2 we introduce the SPDEs with reflecting walls and state the precise conditions on the coefficients. In Section 3 we recall some results on the deterministic obstacle problems which will be used later. In Section 4, we study the skeleton equations and the rate functional. We provide some estimates for the extra measures (local times) in the equation and prove equivalent characterizations of the rate functional. In Section 5, we prove the exponential tightness for the invariant measures. The main result is stated in Section 6. The lower bound of the large deviation is established in Section 7 and the upper bound is obtained in Section 8.

2. Reflected SPDEs

In this section, we introduce reflected stochastic partial differential equations (SPDEs) and state the precise conditions on the coefficients.

Consider the following SPDE with two reflecting walls:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} - \alpha u + f(x, u(x,t)) + \sigma(x, u(x,t))\dot{W}(x,t) + \eta - \xi; \\ \dfrac{\partial u}{\partial x}(0,t) = 0, \qquad \dfrac{\partial u}{\partial x}(1,t) = 0, \quad \text{for } t \geq 0; \\ u(x,0) = u_0(x) \in C([0,1]); \qquad K_1(x) \leq u_0(x) \leq K_2(x), \\ K_1(x) \leq u(x,t) \leq K_2(x), \quad \text{for } (x,t) \in Q, \end{cases} \tag{2.1}$$

here $W(x,t)$ is a space–time Brownian sheet on a filtered probability space $(\Omega, P, \mathcal{F}; \mathcal{F}_t, t \geq 0)$.