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On the asymptotics of locally dependent point processes

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Abstract

We investigate a family of approximating processes that can capture the asymptotic behaviour of locally dependent point processes. We prove two theorems presented to accommodate respectively the positively and negatively related dependent structures. Three examples are given to illustrate that our approximating processes can circumvent the technical difficulties encountered in compound Poisson process approximation (see Barbour and Månsson (2002) [10]) and our approximation error bound decreases when the mean number of the random events increases, in contrast to the increasing of bounds for compound Poisson process approximation.

MSC: primary 60G55; secondary 60E15

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1. Introduction

Random events in space and time often exhibit a locally dependent structure. When the events are very rare and the dependent structure is not too complicated, a natural approach is to declump the events into clusters and then approximate the positions of the clusters by a suitable Poisson process and the sizes of the clusters by independent and identically distributed random elements, as is well documented in [1]. Consequently, compound Poisson and marked Poisson processes are often widely accepted as the 'best approximate models' for clustered rare events.

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The first attempt to estimate the errors of Poisson process approximation seems to go back to [15] with errors measured in the total variation distance, while the errors in the Lévy–Prohorov distance were not studied until [22,25] (see also [32]). All of these studies are based on the stochastic calculus approach with a filtration, a compensator and coupling techniques as the tools used to quantify the distances. Barbour and Brown [4], clearly inspired by the success of Stein's method in multivariate Poisson approximation [3], laid down a general framework for using Stein's method to estimate the Poisson process approximation errors. Their framework can be well adjusted for errors expressed in terms of Janossy densities, Palm distributions and compensators (see [5,34]). In terms of compound Poisson process approximation, there seems to have been no major advance until Arratia et al. [2] replaced the original point process with a new one carrying the information of locations and cluster sizes separately so that the Stein–Chen method for Poisson approximation could be employed to obtain useful error bounds. There are enormous advantages for this approach if one can successfully declump the point process, but the procedure of declumping is far from obvious in applications. By contrast, Barbour and Månsson [10] avoided declumping totally by setting up a framework of Stein's method such that the quality of approximation can be studied directly, and the authors summarized that the direct approach 'has conceptual advantages, but entails technical difficulties' on p. 1492. One of the main difficulties is that Stein's factors, like their counterparts for compound Poisson random variable approximation (see [6,11,12]), are generally too crude to use unless more conditions are imposed such as that the compound Poisson process is very close to a Poisson process. An immediate consequence is that the error bounds obtained often deteriorate when the mean of the point process increases, i.e., more information is available. On the other hand, using the improved estimates for Stein's factors for Poisson process approximation in [34] (cf [16]), Chen and Xia [20] managed to produce error estimates for Poisson process approximation to short range dependent rare events and the estimates will remain small (but not improve either) when the average number of events increases.

It is well-known that the central limit theorem often exhibits the *large sample property*, i.e. the larger the sample size, the better the approximation, as evidenced by the Berry–Esseen bound (see [19]). If we are interested in the total counts of rare and weakly dependent events, the Poisson law of small numbers is the cornerstone of the area. However, the Poisson approximation error does not enjoy the large sample property when more rare events are counted [7]. The shortcoming is due to the fact that a Poisson distribution has only one parameter to fiddle with while a normal distribution has two parameters. When more parameters are introduced, this property can be recovered (see [26,24,18,13,17,28]). In fact, Brown and Xia [17] discovered a large family of distributions that can achieve the same purpose.

The success of compound Poisson process approximation essentially hinges on the fact that the events are very rare. It is tempting to ask whether the approximation theory is still valid when the events are less rare, more heavily dependent and the mean number of events increases. One way to tackle this problem is to keep the approximating process as a Poisson process but weaken the metric for quantifying the difference between point processes [30]. The weaker metric will naturally limit its applicability. The second approach is to introduce more parameters into the approximating point process models. To put the idea into practice, Xia and Zhang [35] introduced a family of point process counterparts of approximating distributions suggested in [17], and named them the polynomial birth–death point processes, or *PBDP* in short. In particular, Xia and Zhang [35] bounded the distance between the Bernoulli process with a constant success probability and a suitable PBDP in terms of the Wasserstein distance (defined in Section 2 below; see also [4]). The assumption of the constant success probability plays the crucial role