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On filtration enlargements and purely discontinuous martingales

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Abstract

Let M be a purely discontinuous martingale relative to a filtration $(\mathcal{F}_t)$. Given an arbitrary extension $(\mathcal{G}_t)$ of the filtration $(\mathcal{F}_t)$, we will provide sufficient conditions for M to be a semimartingale relative to $(\mathcal{G}_t)$. Moreover we describe methods of how to find the Doob–Meyer decomposition with respect to the enlarged filtration. To this end we prove a new and more explicit version of the predictable representation property of Poisson random measures. Finally some concrete examples will show how the method developed may be applied.

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0. Introduction

Whether a stochastic process appears as a semimartingale heavily depends on the filtration that represents the perspective from which the process is observed. If a process is a semimartingale relative to two different filtrations, then in general, the semimartingale decompositions relative to these filtrations will look completely different. The aim of this paper is to contribute to the analysis of how the semimartingale decompositions change if the underlying filtration is *enlarged*. By an *enlargement* of a filtration $(\mathcal{F}_t)$ we mean a filtration $(\mathcal{G}_t)$ containing $(\mathcal{F}_t)$, i.e. a

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filtration satisfying $\mathcal{G}_t \supset \mathcal{F}_t$ for all $t \geq 0$. An introduction into the theory of enlargements of filtrations has been provided in the final chapter of [21]. Other standard references are [12,13].

Well-analysed in the literature has been the question under which conditions *every* $(\mathcal{F}_t)$-semimartingale remains a semimartingale with respect to an enlargement $(\mathcal{G}_t)$. This implication is usually called *hypothesis* (H′). Sufficient and necessary conditions for (H′) to hold are shown in Chapter II in [12]. In the special case of initial enlargements a sufficient condition for (H′) is given in [10].

Many enlargements, however, are such that the semimartingale property is preserved only for a few, but *not* all $(\mathcal{F}_t)$-semimartingales. Thus many authors studied also sufficient conditions for a *given* semimartingale to remain a semimartingale with respect to an enlargement. For that purpose often two special types of enlargements are considered, namely initial and progressive enlargements. In the initial case the original filtration, say $(\mathcal{F}_t)$, is enlarged by a single σ-field $\mathcal{A}$ at all times, and thus the enlargement is given by $\mathcal{G}_t = \mathcal{F}_t \vee \mathcal{A}$. In the progressive case, the enlargement is defined as the smallest expansion of $(\mathcal{F}_t)$ such that a given random time becomes a stopping time.

For initial and progressive enlargements, it is well-known how the semimartingale decompositions with respect to the new filtrations can be obtained. See for instance Chapters III and IV in [12]. Only recent years methods have been developed allowing to derive semimartingale decompositions for alternative types of enlargements. In [4,14] the filtration is enlarged by signals, say on future values of the process, which improve over time. In [3] a representation of the Doob–Meyer decomposition has been obtained for *arbitrary enlargements*, without any assumption on the *kind* of the enlargement.

In [3] only *continuous processes* have been considered. In this paper we will show that the results naturally extend to discontinuous martingales as well, namely to pure jump martingales given as stochastic integrals relative to a compensated Poisson random measure. More precisely, let X be a local martingale of the form

$$X_t = \int_0^t \int_{\mathbb{R}_0} \psi(s,z)\tilde{\mu}(\mathrm{d}s,\mathrm{d}z),$$

where $\tilde{\mu}$ is a compensated Poisson measure and ψ a predictable process such that the stochastic integral with respect to $\tilde{\mu}$ is defined in the usual way (see [11]). We provide sufficient conditions for X to be a semimartingale with respect to an enlarged filtration $(\mathcal{G}_t)$. Moreover, we give explicit descriptions of the semimartingale decomposition. We will only consider filtrations allowing for a semimartingale decomposition of X such that the bounded variation part is absolutely continuous with respect to the Lebesgue measure l. The density of the bounded variation part with respect to l will be called information drift.

On Wiener space one can show with the help of the Clark–Ocone formula that the information drift is the predictable projection of the trace of a Malliavin derivative density (see [9,8] or [1]). In order to obtain a similar representation for pure jump martingales we use a difference operation introduced by Picard [19], and we prove that any bounded random variable F may be written as the stochastic integral of the predictable projection of the Picard difference of F relative to the compensated Poisson random measure. We obtain thus a new and more explicit version of the *predictable representation property*, which corresponds to the Clark–Ocone formula on Wiener space. In the framework of a calculus based on chaotic expansions (which we do not use) similar and related formulas have been obtained for example in [5,16,17,15,6,23].