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Estimation of the volatility persistence in a discretely observed diffusion model

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Abstract

We consider the stochastic volatility model

$$\mathrm{d}Y_t = \sigma_t \, \mathrm{d}B_t,$$

with B a Brownian motion and σ of the form

$$\sigma_t = \Phi\left(\int_0^t a(t,u)\mathrm{d}W_u^H + f(t)\xi_0\right),$$

where W^H is a fractional Brownian motion, independent of the driving Brownian motion B, with Hurst parameter $H \geq 1/2$. This model allows for persistence in the volatility σ. The parameter of interest is H. The functions Φ, a and f are treated as nuisance parameters and ξ_0 is a random initial condition. For a fixed objective time T, we construct from discrete data $Y_{i/n}, i = 0, \ldots, nT$, a wavelet based estimator of H, inspired by adaptive estimation of quadratic functionals. We show that the accuracy of our estimator is $n^{-1/(4H+2)}$ and that this rate is optimal in a minimax sense.

MSC: 60G18; 60G99; 60H05; 60H40; 60G15; 62F12; 62F99; 62M09; 62P05

Keywords: Stochastic volatility models; Discrete sampling; High frequency data; Fractional Brownian motion; Scaling exponent; Adaptive estimation of quadratic functionals; Wavelet methods

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1. Introduction

1.1. Stochastic volatility and volatility persistence

Since the celebrated model of Black and Scholes, the behavior of financial assets is often modelled by processes of type

$$\mathrm{d}S_t = \mu_t\,\mathrm{d}t + \sigma_t\,\mathrm{d}B_t,$$

where S is the price (or the log-price) of the asset, B a Brownian motion and μ a drift process. The volatility coefficient σ represents the fluctuations of S and plays a crucial role in trading, option pricing and hedging. It is well known that stochastic volatility models, where the volatility is a random process, provide a way to deal with the endemic time-varying volatility and to reproduce various stylized facts observed on the markets; see Shephard [34], Barndorff-Nielsen, Nicolato and Shephard [3]. Among these stylized facts, there are many arguments about volatility persistence. This presence of memory in the volatility has in particular consequences for option pricing; see Ohanissian, Russel and Tsay [32], Taylor [35], Comte, Coutin and Renault [11]. Hence, continuous time dynamics have been introduced to capture this phenomenon; see Comte and Renault [12], Comte, Coutin and Renault [11] or Barndorff-Nielsen and Shephard [4]. However, in statistical finance, the question of the volatility persistence has been mostly treated with discrete time models; see among others Breidt, Crato and De Lima [6], Harvey [18], Andersen and Bollerslev [1], Robinson [33], Hurvich and Soulier [22], Teyssière [36]. Concurrently, statistical methods for detecting this volatility persistence have been specifically developed for these models; see Hurvich, Moulines and Soulier [20], Deo, Hurvich and Lu [14], Hurvich and Ray [21], Lee [26], Jensen [24]. In this paper, our objective is to build, for continuous time models, a statistical program allowing us to recover information about the volatility persistence.

1.2. A diffusion model with fractional stochastic volatility

We consider a class of diffusion models whose volatility is a non-linear transformation of a stochastic integral with respect to fractional Brownian motion. Recall that a fractional Brownian motion $(W_t^H, t \in \mathbb{R})$, with Hurst parameter $H \in (0, 1]$, is a self-similar centered Gaussian process with covariance function

$$\mathbb{E}[W_t^H W_s^H] = \frac{1}{2}(|s|^{2H} + |t|^{2H} - |t-s|^{2H}).$$

If $H > 1/2$, the use of fractional Brownian motion (fbm for short) is a way to allow for persistence. Indeed, its increments are then stationary, positively correlated and the value of the Hurst parameter quantifies the presence of so-called long memory between them; see Mandelbrot and Van Ness [27], Taqqu [15]. We define on a rich enough probability space $(\Omega, \mathcal{A}, \mathbb{P})$ a Brownian motion B, a fractional Brownian motion W^H, independent of B, with unknown Hurst parameter $H \in (1/2, 1)$, and a random variable ξ_0, measurable with respect to the sigma algebra generated by $(W_t^H, t \leq 0)$. We fix an objective time $T > 0$ and we consider the one-dimensional stochastic process Y defined by

$$Y_t = y_0 + \int_0^t \sigma_s\,\mathrm{d}B_s, \quad y_0 \in \mathbb{R},\ t \in [0, T], \tag{1}$$