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# Solvability of backward stochastic differential equations with quadratic growth

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**Abstract**

We prove the existence of the unique solution of a general backward stochastic differential equation with quadratic growth driven by martingales. A kind of comparison theorem is also proved.
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## 1. Introduction

In this paper we show a general result of existence and uniqueness of a Backward Stochastic Differential Equation (BSDE) with quadratic growth driven by a continuous martingale. Backward stochastic differential equations have been introduced by Bismut [1] for the linear case as equations of the adjoint process with the stochastic maximum principle. A nonlinear BSDE (with Bellman generator) was first considered by Chitashvili [4]. He derived the semimartingale BSDE (or SBE), which can be considered as a stochastic version of the Bellman equation for a stochastic control problem, and proved the existence and uniqueness of a solution. The theory of BSDEs driven by Brownian motion was developed by Pardoux and Peng [22] for

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more general generators. The results of Pardoux and Peng were generalized by Kobylansky [11], Lepeltier and San Martin [12] for generators with quadratic growth. In the work of Hu et al [8] BMO-martingales were used for BSDEs with quadratic generators in a Brownian setting and in [15–19,21] for BSDEs driven by martingales. By Chitashvili [4], Buckdahn [3], and El Karoui and Huang [7] the well posedness of BSDEs with generators satisfying Lipschitz type conditions was established. Here we suggest a new approach including existence and uniqueness of the solution of the general BSDE with quadratic growth. In the earlier papers [15–20] we studied, like Bobrovnytska and Schweizer [2], the particular cases of BSDEs with quadratic nonlinearities related to the primal and dual problems of mathematical finance. In these works the solutions were represented as a value function of the corresponding optimization problems.

The paper is organized as follows. In Section 2 we give some basic definitions and facts used in what follows. In Section 3 we show the solvability of the system of BSDEs for sufficiently small initial condition and further prove the solvability of one-dimensional BSDEs for arbitrary bounded initial data. At the end of Section 4 we prove the comparison theorem, which generalizes the results of Mania and Schweizer [14], and apply this result to the uniqueness of the solution.

## 2. Some basic definitions and assumptions

Let $(\Omega, \mathcal{F}, \mathbf{F} = (F_t)_{t\geq 0}, P)$ be a filtered probability space satisfying the usual conditions. We assume that all local martingales with respect to $\mathbf{F}$ are continuous. Here the time horizon $T < \infty$ is a stopping time and $\mathcal{F} = F_T$. Let us consider a Backward Stochastic Differential Equation (BSDE) of the form

$$\mathrm{d}Y_t = -f(t, Y_t, \sigma_t^* Z_t)\mathrm{d}K_t - \mathrm{d}\langle N\rangle_t g_t + Z_t^* \mathrm{d}M_t + \mathrm{d}N_t, \tag{2.1}$$

$$Y_T = \xi. \tag{2.2}$$

We suppose that

- $(M_t, t \geq 0)$ is an $R^n$-valued continuous martingale with cross-variations matrix $\langle M\rangle_t = (\langle M^i, M^j\rangle_t)_{1\leq i,j\leq n}$,
- $(K_t, t \geq 0)$ is a continuous, adapted, increasing process such that $\langle M\rangle_t = \int_0^t \sigma_s\sigma_s^* \mathrm{d}K_s$ for some predictable, non-degenerate $n \times n$ matrix $\sigma$,
- $\xi$ is an $\mathcal{F}$-measurable $R^d$-valued random variable,
- $f : \Omega \times R^+ \times R^d \times R^{n\times d} \to R^d$ is a stochastic process such that for any $(y, z) \in R^d \times R^{n\times d}$ the process $f(\cdot, \cdot, y, z)$ is predictable,
- $g : \Omega \times R^+ \to R^{d\times d}$ is a predictable process.

The notation $R^{n\times d}$ here denotes the space of the $n \times d$-matrix $C$ with Euclidean norm $|C| = \sqrt{\mathrm{tr}(CC^*)}$. For some stochastic process $X_t$ and stopping times $\tau, \nu$ such that $\tau \geq \nu$ we define $X_{\nu,\tau} = X_\tau - X_\nu$. For all unexplained notation concerning the martingale theory used below we refer the reader to [9,5,13]. As regards BMO-martingales see [6] or [10].

A solution of the BSDE is a triple $(Y, Z, N)$ of stochastic processes such that (2.1) and (2.2) are satisfied and

- $Y$ is an adapted $R^d$-valued continuous process,
- $Z$ is an $R^{n\times d}$-valued predictable process,
- $N$ is an $R^d$-valued continuous martingale, orthogonal to the basic martingale $M$.

One says that $(f, g, \xi)$ is a generator of BSDE (2.1) and (2.2).

We introduce the following spaces:

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