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Stochastic Processes and their Applications 118 (2008) 1278–1299
stochastic processes and their applications
www.elsevier.com/locate/spa

Diffusion approximation for equilibrium Kawasaki dynamics in continuum

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Received 6 February 2007; received in revised form 20 August 2007; accepted 7 September 2007
Available online 14 September 2007

Abstract

A Kawasaki dynamics in continuum is a dynamics of an infinite system of interacting particles in $\mathbb{R}^d$ which randomly hop over the space. In this paper, we deal with an equilibrium Kawasaki dynamics which has a Gibbs measure μ as invariant measure. We study a diffusive limit of such a dynamics, derived through a scaling of both the jump rate and time. Under weak assumptions on the potential of pair interaction, ϕ, (in particular, admitting a singularity of ϕ at zero), we prove that, on a set of smooth local functions, the generator of the scaled dynamics converges to the generator of the gradient stochastic dynamics. If the set on which the generators converge is a core for the diffusion generator, the latter result implies the weak convergence of finite-dimensional distributions of the corresponding equilibrium processes. In particular, if the potential ϕ is from $C_{\mathrm{b}}^3(\mathbb{R}^d)$ and sufficiently quickly converges to zero at infinity, we conclude the convergence of the processes from a result in [V. Choi, Y.M. Park, H.J. Yoo, Dirichlet forms and Dirichlet operators for infinite particle systems: Essential self-adjointness, J. Math. Phys. 39 (1998) 6509–6536].

MSC: 60F99; 60J60; 60J75; 60K35

Keywords: Continuous system; Diffusion approximation; Gibbs measure; Gradient stochastic dynamics; Kawasaki dynamics in continuum; Scaling limit

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0304-4149/$ - see front matter
doi:10.1016/j.spa.2007.09.001

1. Introduction

A Kawasaki dynamics in continuum is a dynamics of an infinite system of interacting particles in $\mathbb{R}^d$ which randomly hop over the space. The generator of such a dynamics has the form

$$(HF)(\gamma) = -\sum_{x\in\gamma}\int_{\mathbb{R}^d} \mathrm{d}y\, c(\gamma, x, y)(F(\gamma \setminus x \cup y) - F(\gamma)), \quad \gamma \in \Gamma. \tag{1.1}$$

Here, Γ denotes the configuration space over $\mathbb{R}^d$, i.e., the space of all locally finite subsets of $\mathbb{R}^d$, and, for simplicity of notations, we just write x instead of $\{x\}$. The coefficient $c(\gamma, x, y)$ describes the rate at which the particle x of the configuration γ jumps to y.

Let μ denote a Gibbs measure on Γ which corresponds to an activity parameter $z > 0$ and a potential of pair interaction ϕ. In this paper, we will deal with an equilibrium Kawasaki dynamics which has μ as invariant measure. More precisely, we will consider an equilibrium Kawasaki dynamics whose generator (1.1) has the coefficient $c(\gamma, x, y)$ of the form

$$c(\gamma, x, y) = a(x - y) \exp\left[(1/2)E(x, \gamma \setminus x) - (1/2)E(y, \gamma \setminus x)\right]. \tag{1.2}$$

Here, for any $\gamma \in \Gamma$ and $u \in \mathbb{R}^d \setminus \gamma$, $E(u, \gamma)$ denotes the relative energy of interaction between the particle at u and the configuration γ. About the function $a(\cdot)$ in (1.2) we assume that it is non-negative, bounded, has a compact support, and $a(x)$ only depends on $|x|$.

Eq. (1.2) allows the following physical interpretation: particles from γ which have a high relative energy of interaction with the rest of the configuration tend to jump to places where this relative energy will be low, i.e., particles tend to jump from high energy regions to low energy regions.

Note also that the bilinear (Dirichlet) form corresponding to the generator (1.1) and (1.2) admits the following representation:

$$\begin{aligned}\mathcal{E}(F, G) = \frac{z}{2}\int_\Gamma \mu(\mathrm{d}\gamma)\int_{\mathbb{R}^d}\mathrm{d}x\int_{\mathbb{R}^d}\mathrm{d}y\, a(x-y)\exp\left[-(1/2)E(x,\gamma) - (1/2)E(y,\gamma)\right]\\ \times (F(\gamma\cup y) - F(\gamma\cup x))(G(\gamma\cup y) - G(\gamma\cup x)).\end{aligned}$$

Under very mild assumptions on the Gibbs measure μ, it was proved in [11] that there indeed exists a Markov process on Γ with *cádlág* paths whose generator is given by (1.1) and (1.2). We assume that the initial distribution of this dynamics is μ, and perform a diffusive scaling of this dynamics. More precisely, for each $\epsilon > 0$, we consider the equilibrium Kawasaki dynamics whose jump rate is given by formula (1.2) in which $a(\cdot)$ is replaced with the function

$$a_\epsilon(\cdot) := \epsilon^{-d} a(\cdot/\epsilon), \tag{1.3}$$

and we additionally scale time, multiplying it by ϵ^{-2}. We denote the generator of the obtained dynamics by $H^{(\epsilon)}$.

So, the aim of the paper is to show that the scaled dynamics converges, as $\epsilon \to 0$, to a diffusive dynamics on the configuration space Γ. Our main result is that, under weak assumptions on the pair potential ϕ (in particular, we allow ϕ to have a singularity at zero), the generator of the scaled dynamics, $H^{(\epsilon)}$, converges, on a set of smooth local functions, to the generator of the (infinite-dimensional) *gradient stochastic dynamics* (also called *interacting Brownian particles*), see e.g. [1,4,6,12,19–21,25,26] and the references therein. So, the limiting diffusive generator acts as follows: