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Hilbert's objectivity

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Abstract

Detlefsen (1986) reads Hilbert's program as a sophisticated defense of instrumentalism, but Feferman (1998) has it that Hilbert's program leaves significant ontological questions unanswered. One such question is of the reference of individual number terms. Hilbert's use of admittedly "meaningless" signs for numbers and formulae appears to impair his ability to establish the reference of mathematical terms and the content of mathematical propositions (Weyl, 2009/1949; Kitcher, 1976). The paper traces the history and context of Hilbert's reasoning about signs, which illuminates Hilbert's account of mathematical objectivity, axiomatics, idealization, and consistency.

Sommario

Detlefsen (1986) legge il programma di Hilbert come una sofisticata difesa dello strumentalismo, ma Feferman (1998) sostiene che il programma di Hilbert lascia senza risposta alcune significative questioni ontologiche. Una fra queste è il riferimento dei termini individuali numerici. L'impiego da parte di Hilbert di simboli per i numeri e formule per la matematica finistista esplicitamente "privi di senso," sembra impedire la possibilitá di stabilire un riferimento per i termini matematici e un contenuto per le proposizioni (Weyl, 2009/1949; Kitcher, 1976). Questo articolo ripercorre la storia e il contesto del pensiero di Hilbert concernente i simboli; tale contesto getta luce sulla concezione Hilbertiana dell'oggettivitá matematica.

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1. The move to finitism

The history of Hilbert's construction of finitist mathematics has a familiar trajectory. Beginning in 1904 and 1905, in response to external pressures from Brouwer's intuitionism and from Poincaré's objections to his consistency argument based on mathematical induction, Hilbert replaces "contentual" mathematics and physics with formal systems (see Mancosu, 1998a, 1998b; Parsons, 1998; Zach, 1998). In response to the

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larger question of justification, Hilbert takes a number of different positions which culminate in the finitist arguments, according to which the reliability of axiom systems, the justified claim that they will not lead us into error, is sufficient to license their use. Reliability is established through proofs of the consistency of the axiom system.

As Weyl (2009/1949) sums it up, in the process, Hilbert's finitism becomes a "meaningless game" of abstract formula manipulation. Weyl did not mean to be derogatory in his reading. Hilbert himself said that the formulae and "number-signs" of finitist mathematics were without meaning. Still, as Weyl and Kitcher (1976) remark, Hilbert's move to finitist methods appears to undermine his account of the reference of mathematical terms, and in consequence of the truth, or at least the objectivity, of mathematical statements.

What is at issue is whether Hilbert's "number-signs" refer in the same way as do terms of ordinary language that refer to objects or entities. Hilbert's contemporaries Gottlob Frege and Aloys Müller base their objections to Hilbert's methods on the requirement that signs should refer to singular objects, as proper names indicate singular persons or entities. According to Benacerraf (1973), we ought to accept this account of mathematical reference because it mirrors ordinary linguistic usage; the truth conditions for mathematical statements ought not be significantly different from the truth conditions for ordinary propositions. Kitcher (1976) and Zach (1998) respond that this criticism appears unfair to Hilbert's own methods.

Detlefsen (1986) mounts a defense of Hilbert's program on the basis that it is a "philosophically sophisticated and convincing defense of mathematical instrumentalism" (p. x). According to Detlefsen, Hilbert's program supports a "new view of how the ontological commitments of mathematics are to be determined" (p. 2); "the ontological commitments of mathematics are located not in those parts of mathematics which we use to acquire knowledge, but rather in those propositions which are used to establish the reliability of the mathematics thus used" (p. 3). Detlefsen's reading of Hilbert's program is, I believe, broadly in agreement with the account presented here. My focus in the paper is on the more specific question of the relationship between the reference of individual mathematical terms and the objectivity of mathematical claims.

While an instrumentalist or reliabilist reading of Hilbert has been well defended, Hilbert's position still can be criticized from a realist perspective. As Feferman (1998) puts it,

> Hilbert's idea that mathematical concepts "exist" *only* through axiom systems for them is accepted by very few. For, given that the systems we use are necessarily incomplete (granted their consistency), no such system can be said to fully determine its subject matter. So we are led back to philosophical questions about the nature of mathematical concepts and how we come to accept and have our knowledge about them, questions that are just the sort that Hilbert hoped to avoid by his consistency and completeness programs.
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> [pp. 14–15]

While Feferman's point is well taken, he is evaluating whether Hilbert's program can answer questions of contemporary interest. My aim is to describe some of Hilbert's own epistemological concerns, influences, and goals. The effort to do so is rewarded by casting light on why Hilbert himself might have thought that an instrumentalist position on mathematical ontology is defensible; and why it may be defensible now, as a position with limited application.

In the tradition with which Hilbert was engaged, evaluating the truth conditions for ordinary perceptual reports and for the basic statements of physics involved constructing statements using signs. The definition of 'sign' with which Hilbert was most familiar is from the sign and depiction (picture) theories of Hermann von Helmholtz and Heinrich Hertz, respectively. In this tradition, a sign that does not copy properties of an external object can be employed to express thoughts with content within a given framework. Hilbert revises