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ScienceDirect

Historia Mathematica 42 (2015) 155–192

HISTORIA MATHEMATICA

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Figures real, imagined, and missing in Poncelet, Plücker, and Gergonne

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Available online 11 July 2014

Abstract

In the early nineteenth century debate over geometric methodology, Jean-Victor Poncelet characterized pure geometry as reasoning in which the figure is never lost from view. Whether illustrated, described or constructed, Poncelet presented the figure as the primary form of geometrical evidence, a means of justification based in sensory perception. In Poncelet's pure geometry, the objects of geometry were emphatically representational and tangible. By contrast, though classified as analytic geometry, Julius Plücker's contemporary research treated coordinate equations as visual geometric objects—evidence—by focusing on their form and endeavouring to avoid calculations. Working from Poncelet's division between pure and analytic geometries we focus on five versions by three different geometers, of a single conic section construction written between 1817 and 1826. Despite the similarity of their results, Poncelet, Plücker, and Joseph Diaz Gergonne each addressed the problem from contrasting methodological perspectives. We examine how the figure-based distinction materialized in contemporary geometric practices, and what constituted geometric evidence when the figure was lost from view.

Résumé

Dans le débat qui eut lieu au début du dix-neuvième siècle sur la méthodologie de la géométrie, Jean-Victor Poncelet a caractérisé la géométrie pure comme un raisonnement dans lequel la figure n'est jamais perdue de vue. Qu'elle fût illustrée, décrite ou construite, Poncelet présentait la figure comme la forme première de l'évidence géométrique, un moyen de justification fondé sur la perception sensorielle. Dans la géométrie pure de Poncelet, les objets de la géométrie étaient catégoriquement figuratifs et tangibles. D'un autre côté, bien que classées en géométrie analytique, les recherches contemporaines de Julius Plücker traitaient les équations entre coordonnées comme des objets géométriques visuels, en mettant l'accent sur les formes et en s'efforçant d'éviter les calculs. À partir de la division de Poncelet entre géométrie pure et géométrie analytique, nous nous concentrons ici sur cinq versions, écrites entre 1817 et 1826 par trois géomètres, d'une même construction concernant les sections coniques. Malgré la similarité de leurs résultats, Poncelet, Plücker et Joseph Diaz Gergonne s'intéressèrent au problème à partir de perspectives méthodologiques très différentes. Nous examinons ici la manière dont une distinction fondée sur le rôle de la figure s'est concrétisée dans les pratiques géométriques contemporaines et ce qui constitue la preuve géométrique lorsque la figure est perdue de vue.

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http://dx.doi.org/10.1016/j.hm.2014.06.005
0315-0860/

MSC: 01-02; 51-03

Keywords: History of nineteenth century mathematics; Figures; Geometry; Poncelet; Gergonne; Plücker

0. Introduction

Introducing his *Traité des propriétés projectives des figures* in 1822, Jean-Victor Poncelet (1788–1867) explained the practice of ordinary geometry,

> ...la figure est décrite, jamais on ne la perd de vue, toujours on raisonne sur des grandeurs, des formes réelles et existantes, et jamais on ne tire de conséquences qui ne puissent se peindre, à l'imagination ou à la vue, par des objets sensibles ; on s'arrête dès que ces objets cessent d'avoir une existence positive et absolue, une existence physique.[1]
>
> [Poncelet, 1822, xxi]

In this geometry, the figure—a real and existent form composed by sensible objects with positive and absolute physical existence—was central. Moreover, this geometry was a visual, sensible practice in which the figure was never lost from view. Poncelet continued by criticizing the severe rigour of this "restrained" geometry where one must repeat a proof based on whether a given point is to the right or left of a line, and proceeded to promote a new, modern geometry. However, the focus of study in Poncelet's geometry (as his title indicated) continued to be the figure.

The word *figure* itself, did not have a fixed meaning in Poncelet's *Traité*. On the most concrete level, the term *figure* signified the numbered and labelled two-dimensional illustrations often accompanying geometric constructions and definitions, but these particular figures occupied only a fraction of Poncelet's researches. Poncelet discussed "a certain general arrangement of the objects of a figure" (Poncelet, 1822, xii) as well as figures composed of graphic magnitudes [les grandeurs graphiques]. The connection to objects and magnitudes suggest that figures were positional and drawable. However, figures extended well beyond the page. They could be three dimensional, could actively move or deform, could be projected (even projected to infinity), and could contain imaginary points. Figures could be indeterminate or of a particular type, could be primitive or correlative. Reasoning, quantities, expressions, and notions could be figured. Fundamentally, by characterizing ordinary or pure geometry as figure-based, Poncelet intended to contrast it with the equations and calculations of analytic geometry where the figure disappeared.

In all its guises, Poncelet presented the figure as the primary form of geometrical evidence. Following his use of *évidence* and *évident*, most prominently in his *Traité*, we take the term as a means of justification based in sensory perception or as a description of clear mathematics. Poncelet used these terms complementarily, for instance the coincidence of two points was a constructive procedure that could be clearly seen (Poncelet, 1822, 31). Even when this evidence was not supplied by illustrations of figures, the objects of geometry were emphatically representational and tangible. Thus, according to Poncelet, the convolutions of computations found in analytic geometry were hardly bearers of evidence. Working from Poncelet's division between pure and analytic geometries we thus ask how the figure-based distinction materialized in contemporary geometric practices, and what constituted geometric evidence when the figure was lost from view.

[1] "...the figure is described, one never loses it from view, one always reasons about magnitudes, real and existent forms, and never reaches consequences that cannot be painted in the imagination or in sight, by sensible objects; one stops when these objects no longer have a positive and absolute existence, a physical existence."