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Citrabhānu's Twenty-One Algebraic Problems in Malayalam and Sanskrit

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Abstract

This paper studies the Sanskrit and Malayalam versions of Citrabhānu's Twenty-One Problems: a discussion of quadratic and cubic problems from 16th-century Kerala. It reviews the differences in the approaches of the two versions, highlighting the distinction between the Sanskrit indeterminate integer-remainder arithmetical techniques and the Malayali fixed point iterations. The paper concludes with some speculations on the possible transmission of algebraic knowledge between Kerala and the west.

Résumé

Cet article s'occupe des versions Sanskrit et Malayalam de *Vingt et Un Problèmes de Citrabhānu*: une étude des problèmes quadratiques et cubiques provenant de Kerala du 16ème siècle. Nous considérons les différences entre les deux versions, en insistant sur la distinction entre l'arithmétique des entiers dans la version Sanskrit et les itérations de point fixe dans la version Malayalam. L'article se conclut par quelques spéculations sur la possibilité de la transmission du savoir algébrique entre Kerala et l'ouest.

MSC: 01A32; 01A40; 12D10

Keywords: Indian mathematics; Kerala school; Renaissance algebra; Cubic equations

1. Introduction

The object of this paper is an early 16th-century south Indian algebraic text, Citrabhānu's *Twenty-One Problems* (*Ekaviṃśati Praśnottara*), which survives in two versions: one in Malayalam and the other in Sanskrit. The text presents rules for deriving the values of two unknowns, given any pair from the following list: the sum of the two unknowns, their difference, their product, the sum of their squares, the difference

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of their squares, the sum of their cubes, and the difference of their cubes. The title of the text refers to the twenty-one pairs of data that the above seven quantities can yield.

The *Twenty-One Problems* is a product of late medieval Kerala mathematics, an Indian mathematical culture that reached impressive results, and has gained renown for foreshadowing some of the deepest results of early European calculus (see Joseph, 2009a for a general overview). The Sanskrit version of the text is a commentary included in Śaṅkara and Nārāyaṇa's 16th-century commentary on the famous 12th-century Līlāvatī.[1] It is this Sanskrit version that attributes the treatise to the 16th-century Kerala astronomer Citrabhānu (see Joseph, 2009a, p. 21 for brief biographical details). The Malayalam version survives, as far as I know, in a single manuscript included in a bundle of astronomical writings.[2]

It is not clear which of the two versions precedes the other, and whether the Malayalam version should also be attributed to Citrabhānu. A comparison of the rules (see Section 2) seems to fit better with the hypothesis that the Malayalam version represents an earlier stage in thinking about the 21 problems than the Sanskrit one, but I can't rule out the possibility that the Malayalam version is a later simplification of an original closer to the extant Sanskrit source.

The *Twenty-One Problems* continues earlier discussions that concern only the first five quantities above, forming linear and quadratic combinations.[3] The novelty of the treatise is, obviously, the inclusion of cubic sums and differences. Most rules for solving problems related to cubic equations are not presented in what we would consider a closed form; as we will see below, these rules depend on indeterminate integer arithmetic with remainders in the Sanskrit version and on fixed point iterations in the Malayalam one. While they share the same algebraic foundations, the two versions manifest very different kinds of mathematical practice. This and other differences are the focus of this paper.

The next section will compare the indeterminate arithmetic with remainders and the fixed point iteration approaches of the two versions. The third section will discuss other differences in the approaches of the two versions. The fourth section will speculate on the possible connections between the *Twenty-One Problems* and the solution of cubic equations in 16th-century Italy. Selected translations from the Malayalam are included in Appendix A.

2. Sanskrit integer arithmetic vs. Malayalam fixed point iteration

Table 1 summarizes the rules proposed in the two versions for solving the 21 problems. To mitigate our tendency to anachronism, I did not use modern algebraic notation for the unknowns and the seven terms

[1] This commentary was edited by Sarma (1975, pp. 109–129) based on four surviving manuscripts, and the relevant section was analyzed and partially translated by Hayashi and Kusuba (1998) and by Mallayya (2011). A partial Sanskrit copy of the rules and examples (without the solutions of the examples and the commentary, terminating with problem 17) survives as manuscript 22250C in Trivandrum University's Oriental Research Institute Manuscript Library. The palm leaves are partly damaged, and the handwriting is difficult to follow, but the text is almost identical to the relevant portions of Sarma's (1975) edition.

[2] The manuscript, C 541D in Trivandrum University's Oriental Research Institute Manuscript Library (formerly manuscript D 217 in the Madras Government Oriental Manuscripts Library), is mentioned by Sarma (1972, p. 74) and Ulloor (1990, vol. 3, pp. 255–256). The *mana* is indicated as *kudallur meledathe sūryasiddhānta vyākhyā*. The manuscript is relatively well preserved and clearly written. The copyist betrays his weaknesses as a scribe in the opening Sanskrit praise of Gaṇeśa and in the use of Malayali numerals (numerals are missing in several places, sometimes replaced by blank spaces, sometimes miscopied). This might indicate that the scribe was a relative novice, that he was not an experienced mathematician, and/or that he used a corrupt source.

[3] See Hayashi and Kusuba (1998, Section 3) and Kusuba (2010) for Sanskrit references. A Malayalam collection of all ten linear and quadratic problems is available in the Gaṇita Yukti Bhāṣā (Sarma et al., 2008, Chapter 2), but this chapter may postdate our sources. In the Tantrasaṅgraha there is a section dealing with a set of five related trigonometric quantities, showing how to derive any pair of quantities from the other three (Ramasubramanian and Sriram, 2010, 200–228). This section is also entitled "ten problems", but these are not the same problems we are dealing with here. Nevertheless, the dedicatory Sanskrit verse preceding the Malayalam version refers to the ten questions of the Tantrasaṅgraha as its model, and the Malayalam introduction uses a similar combinatorial procedure to count them (see the translation in Appendix A).