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An analysis of errors in mathematical tables

David R. Bellhouse *

Department of Statistical and Actuarial Sciences, University of Western Ontario, London, Ontario, N6A 5B7, Canada

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Abstract

A mode of analysis of the errors in a mathematical table is provided by examining the residuals, the differences between the entries that appear in the table and the appropriate computed values. For the most part, the residuals used here are integer-valued, related to the precision of the table entries. These residuals can provide information about typographical errors, round-off errors, calculation errors and errors due to approximation. Residuals may also be used to make inferences about unknown (to us today) parameters that were used in the construction of a table. The methodology is illustrated by examining four different historical tables.

Résumé

Nous donnons une méthode d'analyse des erreurs dans une table mathématique, fondée sur l'examen des résidus, c'est-à-dire des différences entre les entrées qui apparaissent dans la table et les valeurs calculées. Pour la plupart, les résidus utilisés ici sont des valeurs entières et sont liés à la précision des entrées de la table. Ces résidus peuvent fournir des informations sur les erreurs typographiques, les erreurs d'arrondis, les erreurs de calcul et celles dues à l'approximation. Les résidus peuvent aussi permettre de révéler des paramètres qui nous sont inconnus aujourd'hui mais qui intervenaient dans la construction des tables. Cette méthodologie est illustrée par l'examen de quatre exemples historiques.

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* Fax: +1 519 661 3813.
E-mail address: bellhouse@stats.uwo.ca.

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1. Introduction

Throughout history many mathematical tables have been published. With publication there is always the possibility of numerical errors of one form or another creeping into a table. There are at least five sources of error in mathematical tables: (1) typographical errors in the printed tables; (2) transcription errors in the original source that are copied into the published tables; (3) round-off errors in calculation; (4) calculation errors made by the calculator; and (5) errors due to approximations used by the calculator to ease the burden of calculation. Without the original manuscript or printed source table, transcription errors would be impossible to detect.

The study of these errors, which can be informative, is carried out through the residuals. These are the differences between the tabular values and values that are obtained through correct calculation. More formally, if there are n tabular entries of values t_i for $i = 1, \ldots, n$, the residuals are

$$r_i = t_i - c_i$$

where c_i is the ith value in the table computed by the "correct method". There are two ways in which residuals can be calculated: (1) c_i is calculated to an arbitrary level of precision; and (2) c_i is calculated to the same level of precision as t_i. Approach (1) is current common practice. I will refer to these residuals as "regular" residuals. Under approach (2) the residuals are manipulated by multiplying by the appropriate power of 10 so that the residuals are all integers. For numbers that are not in base 10, other manipulations are appropriate. Sexagesimal numbers, as they appear in Section 5, are multiplied by the appropriate power of 60. This produces what I call "integer-valued" residuals. As will be seen, both regular residuals and integer-valued residuals can be informative in their own ways in the analysis of errors in mathematical tables.

Integer-valued residuals can be used to identify some of the error types listed above. Calculator errors have large residual values with no obvious trend in the digits within each residual. Typographical or transcription errors show up as large residuals in which only one of the digits differs from 0. Round-off errors will have residuals close to 0. The residuals alone may not tell the whole story. Typographical and transcription errors may not be easily separated unless both the printed page and the original source are available.

The use of residuals can go beyond error identification by providing the basis for methods to reconstruct tables originally obtained from a parameter when the original value of the parameter has been lost to us today. Estimating the parameter underlying the structure of an historical table is one aspect of an area generally called "table cracking" [van Dalen, 1989; Van Brummelen and Butler, 1997; Van Brummelen et al., 2013]. In the aspect of table cracking examined here, the researcher is looking for a parameter that in some way minimizes the residuals. I look at how far integer-valued residuals can be taken in this type of analysis compared to the usual use of regular residuals.

In Sections 2, 3 and 4, through integer-valued residuals I illustrate and analyze the types of errors that can occur in printed tables of numbers by examining tables containing entries of sines and logarithms of sines in John Napier's *Mirifici logarithmorum canonis descriptio* [Napier, 1614] and annuity tables in Simon Stevin's *L'Arithmétique* [Stevin, 1585], as well as published tables that are derivative from them. The analysis of Stevin's tables shows the effect of accumulating round-off error. For Napier's tables, the analysis shows the consequences of his method of approximating logarithms. Further, the typographical errors show a trend followed by a random distribution of the errors once the trend is accounted for. Generally, in these three sections I show how analyses of printed tables based on integer-valued residuals can give insight into the printing processes of the day, the arithmetical ability of some of the mathematicians involved, and the nature of approximations that were used.