

Available online at www.sciencedirect.com



HISTORIA MATHEMATICA

Historia Mathematica 41 (2014) 400–437

www.elsevier.com/locate/yhmat

## How algebra spoiled recreational problems: A case study in the cross-cultural dissemination of mathematics

Albrecht Heeffer

Center for History of Science, Ghent University, Faculty of Arts and Letters, LW01, Blandijnberg 2, B-9000, Ghent, Belgium

Available online 27 August 2014

#### Abstract

This paper deals with a sub-class of recreational problems which are solved by a simple memorized rule resulting from an elementary arithmetical or algebraic solution, called proto-algebraic rules. Their recreational aspect is derived from a surprise or trick solution which is not immediately obvious to the subjects involved. Around 1560 many such problems wane from arithmetic and algebra textbooks to reappear in the eighteenth century. Several hypotheses are investigated why popular Renaissance recreational problems lost their appeal. We arrive at the conclusion that the emergence of algebra as a general problem solving method changed the scope of what is considered recreational in mathematics.

© 2014 Elsevier Inc. All rights reserved.

#### Sommario

Questo saggio tratta di una sottoclasse di problemi ricreativi risolti tramite memorizzazione di una semplice regola risultante da una soluzione algebraica o aritmetica, chiamata regola proto-algebraica. L'aspetto ricreativo di questi problemi deriva da una soluzione a sorpresa o da un trucco non immediatamente ovvi ai soggetti coinvolti. Intorno al 1560 svariati problemi di questo tipo sparirono dai manuali di algebra e aritmetica, per riapparire nel diciottesimo secolo. Diverse ipotesi sono vagliate sul perché problemi ricreativi popolari nel Rinascimento persero attrattività, per giungere alla conclusione che l'emergere dell'algebra come metodo generale di risoluzione di problemi cambiò la portata di ciò che era considerato ricreativo in matematica. © 2014 Elsevier Inc. All rights reserved.

MSC: 00A08; 97A20

Keywords: Recreational; Problems; Algebra; Cross-cultural

http://dx.doi.org/10.1016/j.hm.2014.06.004 0315-0860/© 2014 Elsevier Inc. All rights reserved.

E-mail address: albrecht.heeffer@ugent.be.

### 1. Introduction

Anyone engaging in a study of recreational mathematics soon discovers that many of the problems that are still popular today go back a long time in history. In a recent book on the mathematics behind card tricks by two professional mathematicians (Diaconis and Graham, 2012, 106–114) the authors traced the history of a popular three-object divination problem to Prevost (Prevost, 1584) and Bachet (Bachet, 1612). However, after twenty years of research they were surprised to learn that the problem is a frequently recurring one in abbaco manuscripts of the *quattrocento* and appears in even earlier sources.

Equally surprising is the fact that the same or similar problems appear in very different cultures and geographical regions. One such example is the sliding ladder problem about a ladder of a given length which stands against a wall and which is moved from its original position on the ground. It features in several Old-Babylonian tablets BM 85196 (Høyrup, 2007, 275-6), BM 34568 (Friberg, 1981, 307-8), in Egyptian papyri, Cairo JdE. 89127-30 (Parker, 1972, 13-43), in Chinese classics, Jiǔ zhāng suàn shù 九章算術, Nine chapters of the mathematical art (Chemla and Shuchun, 2004), in Sanskrit mathematical texts, Bhāskara I. 629. Commentary on the Aryabhatīya, II.16 (Keller, 2006, 79–83) and also in the works by Bhāskara II, in Arabic texts, Hasib Tabarī, Miftāh al-muʿāmalāt (Bagheri, 1999), in many abbaco manuscripts, e.g. Gherardi, Libro di ragioni (Florence, BNCF, Magl. Cl. XI, 86; Arrighi, 1987b) as well as in Fibonacci's De Practica Geometrie (Hughes, 2008, 77) and Pacioli's Summa (Pacioli, 1494, part 2, ff. 54<sup>v</sup>-55<sup>v</sup>), and in seventeenth century works on recreational mathematics (van Etten, 1624, prob. 89; Ozanam, 1725, 320-1, prob. 42). The interest in this specific problem thus not only spans a period of more than three millennia but also five very different cultures and mathematical practices! Though conceptualization, contextual meaning and solution methods may differ amongst these cultures, at least we can induce from this example that problems we now consider as recreational mathematics are omnipresent in mathematical practice and that these travel easily between different cultures.

One of the reasons for the multi-cultured aspect of recreational mathematics is that mathematical knowledge is often embedded in folk stories, riddles, tricks, tangible practices which form the basis for some types of recreational problems. Solution methods, rules and mnemonic devices are in concert with the problems. Many recreational problems have been disseminated as folk stories through merchant connections and trade routes. Embedding mathematics in cultural practices which can be adapted to suit the cultural context allows problems to cross cultural boundaries. That is the reason why the same problems turn up in such diverse cultures.

This paper presents a case study on a sub-class of recreational problems which are based on some elementary arithmetical or algebraic solution. Their recreational aspect stems from a surprise or trick solution which is not immediately obvious to the subjects involved. The legacy problem about a dying father with an unknown number of children (discussed below) is a challenge to solve unless you know the simple rule of thumb which gives you the answer to this and similar problems. The first book to coin the term recreational mathematics in the title (van Etten, 1624) often uses the qualification "wonder to those that are ignorant in the cause".<sup>1</sup> As in divination problems, it is important to conceal the mathematical principles behind the trick or problem to make it surprising and appealing. The title of this paper somewhat polemically states that this recreational aspect of problems disappears when a general solution method – as is algebra – is applied to solve such problems. I will indeed demonstrate that this is the case for some specific problems, while other types of recreational problems are less exposed to algebraic solutions.

I will first discuss the practical context of arithmetical problems. Renaissance recreational problems are often situated in a practical context to give them some flair or alleged utility. In the next part it is shown

<sup>&</sup>lt;sup>1</sup> I have previously challenged the attribution of this book to the Jesuit Jean Leurechon, and proposed the printer/engraver Jean Appier Hanzelet as the compiler of the problems (Heeffer, 2006b). For further discussion on the authorship see my forthcoming critical edition (Heeffer, forthcoming).

Download English Version:

# https://daneshyari.com/en/article/1158406

Download Persian Version:

https://daneshyari.com/article/1158406

Daneshyari.com