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"Voici ce que j'ai trouvé:" Sophie Germain's grand plan to prove Fermat's Last Theorem

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Abstract

A study of Sophie Germain's extensive manuscripts on Fermat's Last Theorem calls for a reassessment of her work in number theory. There is much in these manuscripts beyond the single theorem for Case 1 for which she is known from a published footnote by Legendre. Germain had a full-fledged, highly developed, sophisticated plan of attack on Fermat's Last Theorem. The supporting algorithms she invented for this plan are based on ideas and results discovered independently only much later by others, and her methods are quite different from any of Legendre's. In addition to her program for proving Fermat's Last Theorem in its entirety, Germain also made major efforts at proofs for particular families of exponents. The isolation Germain worked in, due in substantial part to her difficult position as a woman, was perhaps sufficient that much of this extensive and impressive work may never have been studied and understood by anyone.
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Résumé

Une étude approfondie des manuscrits de Sophie Germain sur le dernier théorème de Fermat, révèle que l'on doit réévaluer ses travaux en théorie des nombres. En effet, on trouve dans ses manuscrits beaucoup plus que le simple théorème du premier cas que Legendre lui avait attribué dans une note au bas d'une page et pour lequel elle est reconnue. Mme Germain avait un plan très élaboré et sophistiqué pour prouver entièrement ce dernier théorème de Fermat. Les algorithmes qu'elle a inventés sont basés sur des idées et resultats qui ne furent indépendamment découverts que beaucoup plus tard. Ses méthodes sont également assez différentes de celles de Legendre. En plus, Mme Germain avait fait de remarquables progrès dans sa recherche concernant certaines familles d'exposants. L'isolement dans lequel Sophie Germain se trouvait, en grande partie dû au fait qu'elle était une femme, fut peut-être suffisant, que ses impressionnants travaux auraient pu passer complètement inaperçus et demeurer incompris.
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1. Introduction

Sophie Germain (Fig. 1)[1] was the first woman known for important original research in mathematics.[2] While perhaps more famous for her work in mathematical physics, which earned her a French Academy prize, Germain is also credited with an important result in number theory toward proving Fermat's Last Theorem. We will make a substantial reevaluation of her work on the Fermat problem, based on translation and mathematical interpretation of numerous documents in her own hand, and will argue that her accomplishments are much broader, deeper, and more significant than has been realized.

Fermat's Last Theorem refers to Pierre de Fermat's famous 17th-century claim that the equation $z^p = x^p + y^p$ has no natural number solutions x, y, z for natural number exponents $p > 2$. The challenge of proving this assertion has had a tumultuous history, culminating in Andrew Wiles' success at the end of the 20th century [Ribenboim, 1999, XI.2].

Once Fermat had proven his claim for exponent 4 [Dickson, 1920, 615ff; Weil, 1984, 75ff], it could be fully confirmed just by substantiating it for odd prime exponents. But when Sophie Germain began working on the problem at the turn of the 19th century, the only prime exponent that had a proof was 3 [Dickson, 1920, XXVI; Edwards, 1977, Ch. 3; Ribenboim, 1999, I.6, IV; Weil, 1984, 335ff]. As we will see, Germain not only developed the one theorem she has long been known for toward proving part of Fermat's Last Theorem for all primes. Her manuscripts reveal a comprehensive program to prove Fermat's Last Theorem in its entirety.

1.1. Germain's background and mathematical development

Sophie Germain[3] was born on April 1, 1776 and lived with her parents and sisters in the center of Paris throughout the upheavals of the French Revolution. Even if kept largely indoors, she must as a teenager have heard, and perhaps seen, some of its most dramatic and violent events. Moreover, her father, Ambroise-François Germain, a silk merchant, was an elected member of the third estate to the Constituent Assembly convened in 1789 [Bucciarelli and Dworsky, 1980, 9ff]. He thus brought home daily intimate knowledge of events in the streets, the courts, etc.; how this was actually shared, feared, and coped with by Sophie Germain and her family we do not know.

Much of what we know of Germain's life comes from the biographical obituary [Libri, 1832a] published by her friend and fellow mathematician Guglielmo Libri, shortly after her death in 1831. He wrote that at age 13, Sophie Germain, partly as sustained diversion from her fears of the Revolution beginning outside her door, studied Montucla's *Histoire des*

[1] From Bucciarelli and Dworsky [1980, 17].

[2] A biography of Germain, with concentration on her work in elasticity theory, discussion of her personal and professional life, and references to the historical literature about her, is the book [Bucciarelli and Dworsky, 1980].

[3] Much of our description here of Germain's background appears also in Pengelley [in press].

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