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Beyond Cartesian limits: Leibniz's passage from algebraic to "transcendental" mathematics

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Abstract

This article deals with Leibniz's reception of Descartes' "geometry." Leibnizian mathematics was based on five fundamental notions: calculus, characteristic, art of invention, method, and freedom. On the basis of methodological considerations Leibniz criticized Descartes' restriction of geometry to objects that could be given in terms of algebraic (i.e., finite) equations: "Descartes's mind was the limit of science." The failure of algebra to solve equations of higher degree led Leibniz to develop linear algebra, and the failure of algebra to deal with transcendental problems led him to conceive of a science of the infinite. Hence Leibniz reconstructed the mathematical corpus, created new (transcendental) notions, and redefined known notions (equality, exactness, construction), thus establishing "a veritable complement of algebra for the transcendentals": infinite equations, i.e., infinite series, became inestimable tools of mathematical research.

Zusammenfassung

Der Aufsatz behandelt Leibniz' Aufnahme von Descartes' „Geometrie". Die Leibnizsche Mathematik war auf fünf grundlegenden Begriffen aufgebaut: Kalkül, Charakteristik, Erfindungskunst, Methode, Freiheit. Leibniz' methodologische Betrachtungen zogen seine Kritik der cartesischen algebraischen Methoden nach sich, die das Gebiet der Geometrie definierten: „Descartes' Geist war die Grenze der Wissenschaft". Die Unvollkommenheit der Algebra (Lösung algebraischer Gleichungen höheren Grades) ließ Leibniz lineare Algebra entwickeln und eine Wissenschaft des Unendlichen entwerfen. Leibniz baute also das Gebäude der Mathematik neu auf, schuf neue Begriffe (transzendent) und definierte bekannte Begriffe neu (Gleichheit, Genauigkeit, Konstruktion). Auf diese Weise begründete er eine „wahre Ergänzung der Algebra für transzendente Größen": unendliche Gleichungen, das heißt unendliche Reihen, wurden unschätzbare Hilfsmittel der mathematischen Forschung.

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Introduction: sources

Since 1976, seven volumes have appeared, comprising about three thousand printed pages of Leibnizian mathematical studies. They partly, mainly, or exclusively deal with algebra or related topics:

[Leibniz, 1976a]: combinatorics (340 pp.),
[Leibniz, 1976b]: arithmetic, algebra (200 pp.),
[Knobloch, 1980]: determinant theory (330 pp.),
[Leibniz, 1990]: geometry, number theory, algebra (954 pp.),
[Leibniz, 1993]: infinite series (160 pp.),
[Leibniz, 1996]: algebra (870 pp.),
[Leibniz, 2003]: infinite series and sequences (880 pp.).

These writings support by documentary evidence Leibniz's overwhelming interest in algebra and its relation to geometry.

The present article is based on these new, available sources.[1] It tries to show in what ways Leibniz's contributions to linear algebra and his understanding of Descartes' *Geometry* were motivated by imperfections in algebra and were profoundly influenced by Leibniz's new conception of mathematics.

1. From the theory of equations to linear algebra

In 1924, the French poet Paul Valéry praised the algebraic use of unknowns in the following way:

> Quelle idée plus digne de l'homme que d'avoir nommé ce qu'il ne sait point ? Je pus engager ce que j'ignore dans les constructions de mon esprit, et faire d'une chose inconnue une pièce de la machine de ma pensée. [Valéry, 1999]

Yet, in Leibniz's eyes algebra suffered from two imperfections:

(1) The algorithmic solution of the general algebraic equation of nth degree was still unavailable;
(2) Algebraic equations did not suffice to comprehend transcendental problems in geometry.

Like all of his contemporaries, Leibniz was convinced of the solvability of the first problem. His own attempts in this direction resulted in the emergence of determinant theory. His studies of transcendental

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[1] The authors of the monograph *The Beginning and Evolution of Algebra* [Bashmakova and Smirnova, 2000] did not take notice of these volumes. Their only reference to Leibniz concerns his well-known letter to l'Hospital dating from 1693 and published in 1850 [Leibniz, 1850, pp. 236–241], in which Leibniz derived the solution of a system of three linear equations. In this respect, the monograph represents the state of affairs of 1850.