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ScienceDirect
Historia Mathematica 36 (2009) 247–272
HISTORIA MATHEMATICA
www.elsevier.com/locate/yhmat

A proportional view: The mathematics of James Glenie (1750–1817)

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Available online 12 February 2009

Abstract

The mathematical work of James Glenie (1750–1817) was published at irregular intervals during a turbulent life. His ideas, mostly deriving from his time as an Assistant in Mathematics at St Andrews University in Scotland, were developed intermittently over a period of thirty-seven years. His mathematical achievements, underestimated by previous historians, were deeply rooted in Euclidean geometry and his own generalized theory of proportion. Among them are many new geometrical constructions and proofs, a novel demonstration of the binomial theorem, and an alternative approach to the differential calculus.

Résumé

La vie de James Glenie (1750–1817) était turbulente, et ses ouvrages mathématiques furent publiés irrégulièrement. Ses idées, prenant leur origine, par la plupart, dans la periode où il était Assistant en mathématique à l'Université de St Andrews en Écosse, furent developpées pendant trente-sept ans environ. Ses succès mathématiques, sousestimés auparavant, furent enracinés dans la géometrie Euclidienne et dans sa théorie originale des proportions géneralisées. Parmi eux il y a plusieurs constructions et preuves géometriques nouvelles, une démonstration originale du théorème binomial, et une approche alternative au calcul différentiel.

MSC: 01A50; 01A55; 01A70; 4003; 5103

Keywords: Euclidean geometry; Theory of proportion; Binomial theorem; Calculus; St Andrews; Canada

1. Introduction

The life and works of James Glenie (1750–1817) have been considered by just a few historians. After study at St Andrews University in Scotland, and brief employment there as an Assistant in Mathematics, his turbulent life encompassed service in Canada as a military engineer and then radical politician, controversies in England over planned sea defences, and involvement in various legal proceedings. In his own day, Glenie was a quite well-known and even notorious figure. An account of his life is given in the next section, where references to fuller historical studies are given. The remaining sections focus on his mathematical writings. Short summaries of his publications on gunnery, fortification, and mathematics have earlier appeared in Goodwin [1889]; Roberts [1970]; Johnson [1997, 1998, 2004]. Johnson deserves credit for bringing the work of Glenie to the attention of modern readers: his papers contain much information, but also a few inaccuracies, and his discussion of Glenie's mathematics is not profound.

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0315-0860/$ – see front matter
doi:10.1016/j.hm.2008.12.001

During his eventful and unquiet life, it is surprising that Glenie retained his love of mathematics, and intermittently published works of considerable interest. Though it seems that he made no great attempt to "keep up with the subject," he continued to deploy the skills acquired during his days as a student and assistant at St Andrews University in the 1770s. These centered mainly on Euclidean geometry and the theory of proportion, which he applied and extended in unexpected ways. His geometrical constructions and proofs established many new results and proved old ones left unproven by Matthew Stewart. His *Doctrine of Universal Comparison* [1789] and his *Antecedental Calculus* [1793], though unwieldy, gave a workable alternative approach to the differential calculus that was more firmly founded than modern writers have allowed. And his Euclidean demonstration of the binomial theorem [Glenie, 1799], for arbitrary real powers, deserves recognition as perhaps the earliest attempt that *almost* stands up to modern scrutiny.[1]

The best-known among these little-remembered publications is Glenie's *Antecedental Calculus*. Charles Hutton's *A Mathematical and Philosophical Dictionary* of 1796 describes this as "a branch of general geometrical proportion, or universal comparison ... invented by Mr. James Glenie and published by him in 1793" [Hutton, 1796, 1: 121].[2] In 1919, Florian Cajori discussed Glenie's work in a chapter devoted to "abortive attempts at arithmetisation" of the calculus, all of which he deemed "either a complete failure or so complicated as to be prohibitive." Implying that Glenie's work belongs to the latter category, he rightly observed that it "plays no part in the later history of fluxions" and that "the style of exposition is poor" [Cajori, 1919, 237–238]. Glenie's calculus is also briefly discussed by Guicciardini [1989, 104], who comments on the "obvious weakness of Glenie's mathematics" in simply deleting higher-order powers of small increments in order to obtain his equivalent of Newton's fluxions.

It is the contention of this paper that the mathematical achievements of James Glenie have been underestimated. No doubt, these played little part in the subsequent development of mathematics; and his mathematical language, based on Euclidean geometry and the theory of proportion, was already falling from fashion in his own lifetime. The unfavorable judgment of Guicciardini is based, I believe, on incomplete readings of Glenie's publications; but Cajori's criticism of the complexity and near-unreadability of some of Glenie's works is well founded. Nevertheless, these works repay close study, for he employed his traditional tools with insight and originality to establish many new results.

Many of Glenie's publications were privately printed pamphlets, with only a few in established journals. Just two papers were published in each of the *Philosophical Transactions of the Royal Society of London* and the *Transactions of the Royal Society of Edinburgh*, and a few pieces appeared in Maseres' eccentric *Scriptores Logarithmici* and in the *Ladies' Diary*. Glenie's main papers on his Universal Comparison and Antecedental Calculus and his several papers on fortification all appeared as pamphlets, and some work remained unpublished in manuscripts that survive in the archives of the Royal Society of London. It may be significant that no papers by Glenie were published in journals of the Royal Society of London after he quarrelled spectacularly with its President in 1784. But he certainly remained involved, for he presented copies of his pamphlets to the Society, and some of his works in manuscript seem to have been read at Society meetings, though not published.

Glenie's first three scientific publications appeared in 1775–1777: two papers in *Philosophical Transactions of the Royal Society of London*, and his 163-page *A History of Gunnery*.[3] It has been suggested that Glenie's interest in gunnery may have been stimulated by Charles Hutton at Woolwich. However, Glenie's work was published in Edinburgh in 1776 and, though dedicated to Viscount Townshend, Master Gunner at the Ordnance, it was surely written while he was working in St Andrews as assistant to the Professor of Mathematics, Nicolas Vilant.

The recurrent themes of Glenie's mathematical publications are new theorems in Euclidean geometry and his "Doctrine of Universal Comparison or General Proportion." The latter involved his own algebra of ratios, by which *ratios of ratios* were manipulated according to various rules. He had begun this work on proportion and also his related "Antecedental Calculus" by 1774, while working as Vilant's assistant. In 1793, the latter calculus was published as a short 16-page tract in continuation of his work on general proportion. Having just been elected a fellow of the Royal Society of Edinburgh, in 1794 Glenie prepared a further paper on the principles of his Antecedental Calculus for that society's *Transactions*. This Antecedental Calculus was Glenie's bold attempt to supplant Newton's fluxionary calculus by one

[1] The next such proof of which the present author is aware dates from 1809, and is due to another Scot, William Spence, who employed more modern analytical methods [Spence, 1809, v–viii].

[2] Hutton and Glenie were friends and associates, with common interests in ballistics and mathematics. Also, during 1773–1818, Hutton edited the *Ladies' Diary* to which Glenie was a contributor.

[3] A full list of Glenie's publications is given in the References at the end of this paper.